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THEORY AND PRACTICE OF THE TESTING OF GAS WELLS

JULY 1965

OIL AND GAS CONSERVATION BOARD

603 SIXTH AVENUE SOUTH WEST • CALGARY, ALBERTA

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SECOND EDITION

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PRICE $2.50 (Sales Tax Included)

Preface to the Second Edition

This second edition of the Theory and Practice of the Testing of Gas Wells has resulted from a detailed review of the first or Discussion Draft carried out during 1964 and 1965. The Board requested the view of the Alberta oil and gas industry on the Discussion Draft through the Alberta Division of the Canadian Petroleum Association. The Association established a special committee, under the chairmanship of Mr. R. L. Taylor of Shell Canada Limited, to review the document and the procedures proposed in it. The other committee members were G. C. Whittaker, Pacific Petroleums Ltd., V. P. Peters, Pan American Petroleum Corporation, J. Hnatiuk, The British American Oil Company Limited, H. S. Simpson, Imperial Oil Limited, and S. M. Thorne, Socony Mobil Oil of Canada, Ltd. This group spent many hours in a detailed study of the Discussion Draft and made many useful suggestions which have been incorporated in the Second Edition. The Board acknowledges this valuable contribution.

In addition, Dr. D. L. Katz of the University of Michigan, Mr. K. Aziz of Rice University and the late Mr. F. K. Beach, Petroleum Consultant, were good enough to send constructive comments for incorporation into the Second Edition. These also are gratefully acknowledged.

Mr. G. J. DeSorcy, Chief Gas Engineer for the Board, accepted the major responsibility for preparing the revisions with Mr. A. S. Telford of the Board's Gas Department giving able assistance.

In addition to introducing the suggestions of the aforementioned, Mr. DeSorcy has made a number of revisions of his own and as suggested by other members of the Board organization.

The Board has been pleased with the acceptance of the Discussion Draft by industry and believes that this Second Edition, incorporating the suggestions received from industry, should serve the objective originally set - that of improving engineering practice in the testing of gas wells and aiding in conservation.

G. W. Govier
Chairman
Oil and Gas Conservation Board

June, 1965

Preface to February 1964 Edition

This manual on the Theory and Practice of the Testing of Gas Wells has been prepared to serve a growing need in the Province of Alberta. It is in what might be called "discussion draft" form. It is hoped that suggestions for its improvement will be received from industry during 1964 and it is planned that a revised and improved version will be published in 1965.

Many advances have been made in the understanding both of the reservoir and the well bore flow of natural gas since the publication of the well known "Monograph 7" of the U.S. Bureau of Mines in 1937. Few of these developments, however, have been incorporated into routine well testing procedures and the engineer who is not a specialist in the field may not be familiar with the technical literature describing them.

The manuals published by the Texas Railroad Commission, the Kansas State Corporation Commission, and recently by the Interstate Oil Compact Commission have contributed greatly to the precision of definition of certain of the tests and to a standardization of methods of calculation. These publications, however, were not intended to include reviews or criticism of the pertinent theory and except for the last mentioned they deal only with "stabilized flow" testing.

One of the most significant improvements in the scientific understanding of gas well testing is in connection with the unsteady state or unstabilized flow behavior. It seems appropriate to take advantage of this and other developments and to design well testing procedures accordingly.

The preparation of the first draft of this manual was carried out in 1958-59 under my guidance as a class project in the graduate course, Advanced Natural Gas Engineering, at the University of Alberta, Edmonton, by graduate students: D. Batcheller, T. Fekete, I. Nielsen, C. Winter and

S. Qayum. Mr. F. Werth, Instructor in Petroleum Engineering, assisted in the preparation of certain of the tables and figures. Valuable assistance also was given at this time by Mr. J. G. Stabback, then Chief Gas Engineer of the Oil and Gas Conservation Board, and by representatives of the Alberta gas industry, including The British American Oil Company Limited, Northwestern Utilities Limited, Pacific Petroleums Limited, Shell Canada Limited and Imperial Oil Limited. Some of these representatives attended class discussions and many made data from company files available.

The first draft was reworked and revised chiefly by Messrs. J. Pletcher, Assistant Gas Engineer and Mr. S. A. Qayum, Temporary Assistant Gas Engineer of the staff of the Oil and Gas Conservation Board. Mr. K. Aziz, Assistant Professor of Petroleum Engineering, University of Alberta, also gave assistance at this time. This work was carried out intermittently as time permitted during 1961 and 1962. Finally, in mid-1963, Mr. G. J. DeSorcy, Assistant Chief Gas Engineer of the Board was assigned the full time task of preparing a draft suitable for publication and distribution to industry. Mr. DeSorcy and I altered the emphasis of the early draft from one directed to flow tests and related bottom hole pressure calculations to the broader one of the present treatment. Others of the staff of the Board have participated and shared in the checking of calculations.

The manual is being released in its present form not because it is thought to be a finished product but because the Board believes that it will even now be of some value to industry in improving the practice of testing gas wells and, therefore, serve the interests of conservation. Also, in this way, the Board hopes that it will receive suggestions from industry for improvement of the manual.

The manual suggests certain procedures, those which the Board

believes best, but it is not intended as a directive with respect to procedures. Following its revision in 1965, however, certain sections of it may be considered suitable for incorporation, by reference, into regulations issued under The Oil and Gas Conservation Act.

G. W. Govier, P. Eng.
Chairman
Oil and Gas Conservation Board

February, 1964

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The first [illegible]

[illegible] Assistant [illegible]

[illegible] of the [illegible]

[illegible] Assistant Professor [illegible]

[illegible]

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NOMENCLATURE

a	=	$\dfrac{1.418 \times 10^{6}\, \mu_a Z_a T_a \ln (r_f/r_s)}{kh}$	(by definition)
A	=	cross sectional area of flow, square cm.	
AOF	=	absolute open flow.	
b	=	$\dfrac{3.14 \times 10^{-6}\, \beta\, G\, Z_a T_a}{h^2}\left(\dfrac{1}{r_s} - \dfrac{1}{r_f}\right)$	(by definition)
C	=	back pressure equation coefficient.	
d	=	inside diameter of flow pipe, inches.	
D_{eff}	=	effective flow diameter of annular space, feet = $D_2 - D_1$.	
D_1	=	outside diameter of tubing, feet.	
D_2	=	inside diameter of casing, feet.	
D	=	inside diameter of flow pipe, feet.	
D_g	=	average grain diameter, feet.	
e	=	base of naperian logarithm.	
f	=	friction factor.	
f_{CK}	=	modified friction factor (Cornell and Katz).	
f_{FLB}	=	modified friction factor (Fancher, Lewis and Barnes).	
F^2	=	$\dfrac{2.6665\, f\, Q^2}{d^5}$	(by definition)
F_t	=	turbulence contribution factor.	
dF	=	energy losses due to irreversibilities.	
g_c	=	dimension conversion factor.	
G	=	specific gravity of gas.	
G_c	=	specific gravity of separator condensate.	

G_m = specific gravity of gas resulting from combination of separator products.

G_s = specific gravity of separator gas.

h = pay thickness, feet.

k = permeability, millidarcys.

K = permeability, darcys.

K_i = vapor - liquid equilibrium ratio for component i.

L = length, feet.

q_D = dimensionless flow rate.

M = molecular weight of gas, pounds per pound mole.

n = reciprocal slope of the back pressure curve.

N_{Re} = Reynolds number.

N_{ReCK} = modified Reynolds number (Cornell and Katz).

N_{ReFLB} = modified Reynolds number (Fancher, Lewis and Barnes).

P = pressure, psia.

ΔP = pressure drop, lbs. per sq. ft.

P_a = average pressure, psia.

P_c = critical pressure, psia.

P_D = dimensionless pressure = $\left(\frac{P}{P_f}\right)^2$ (by definition)

P_f = shut-in formation pressure, psia.

P_m = pressure measured during a build-up or drawdown test, psia.

P_o = standard pressure = 14.65 psia in Alberta.

P_p = pressure at time of previous survey, psia.

P_r = reduced pressure.

P_{ra} = reduced average pressure.

P_{rs} = reduced sandface pressure.

P_{rw} = reduced wellhead pressure.

P_s = flowing sandface pressure, psia.

P_t = dimensionless drawdown.

P_w = wellhead pressure, psia.

P^* = false value of final static pressure, psia.

q = volume flow rate, cu. cm. per sec.

Q = gas flow rate, millions of cubic feet per day at standard conditions.

r = radius, feet.

r_a = apparent or steady-state radius of drainage, feet.

r_D = dimensionless radius = $\frac{r}{r_s}$ (by definition)

r_f = exterior boundary radius, feet.

r_s = effective radius of well bore, feet.

R = gas law constant.

R_c = gas-condensate ratio, cubic feet per barrel.

S = $\frac{2GL}{53.34\ T_a Z_a}$ (by definition)

t = time, hours.

t_D = dimensionless time.

t_f = time of flow, hours.

t_s = time to stabilization, hours.

Δt = shut-in time, hours.

T = temperature, $^\circ$R.

T_a = average temperature, $^\circ$R.

T_c = critical temperature, $^\circ$R.

T_o = standard temperature = 520°R.

T_r = reduced temperature.

T_{ra} = reduced average temperature.

T_{rs} = reduced sandface temperature.

T_{rw} = reduced wellhead temperature.

T_s = sandface or formation temperature, ^oR.

T_w = wellhead temperature, ^oR.

u = variable related to reservoir and flow characteristics.

v = specific volume of gas, cubic feet per pound.

V = velocity of flow, feet per second.

V_a = apparent velocity, feet per second.

V_c = condensate vaporizing volume ratio, cubic feet per barrel.

x_i = mole fraction of component i in liquid phase.

y_i = mole fraction of component i in vapor phase.

$y(u)$ = a function of u.

Z = gas compressibility factor.

Z_a = average compressibility factor.

Z_s = sandface compressibility factor.

Z_w = wellhead compressibility factor.

β = turbulence factor, centimeters^{-1}.

ρ = density of gas, lbs-mass per cubic feet.

ϕ = gas filled porosity, fraction.

δ = absolute roughness of pipe.

δ/d = relative roughness of pipe.

μ = viscosity, centipoise (at any pressure and temperature).

μ_1 = viscosity, centipoise (at one atmosphere pressure and any temperature)

μ_a = average viscosity, centipoise.

μ' = viscosity of gas, lbs-mass per foot second.

1. INTRODUCTION

The ability to analyse the performance and forecast the productivity of gas wells with a reasonable degree of accuracy is of utmost importance in today's natural gas industry. One of the most useful aids in analysing gas well performance is the flowing well test. The results of such tests are often used by regulatory bodies in setting maximum gas withdrawal rates. They are also employed by producing and transportation companies in projecting gas well deliveries.

A complete analysis and understanding of the results of an appropriate well test enables one to determine the rate at which a well will flow against a particular pipeline "back" pressure, and also to predict the manner in which the flow rate will decline with depletion and the resulting drop in reservoir pressure. This type of projection is necessary in the preparation of field development programs, the design of gathering and pipeline facilities as well as processing plants and is often used in the negotiation of gas sales contracts.

Other important applications of the well test and information gathered during testing are in the estimation of gas reserves associated with a well or group of wells and in making various types of special reservoir studies.

History

The maximum theoretical delivery rate of a gas well has been described as the absolute "open flow" potential, which in a limited way is still a useful concept. The absolute open flow potential is defined as the ability of a well to produce against zero sandface back pressure.

In the early days of the industry the maximum potential was directly measured by gauging the flow of gas from a well which had been opened to the atmosphere. Opening a well to atmosphere, however, results in a flow rate which is not precisely equivalent to the absolute open flow potential, i. e. the flow that would take place against a zero sandface back pressure. This maximum flow rate to the atmosphere at the surface is often termed the practical open flow potential and for shallow wells is very close to the absolute open flow potential. On the other hand, for deep and high capacity wells (where the frictional resistance to the flow of the gas is considerable) the difference between these flow potentials is often very significant.

The field gauging of open flow potentials often did not give reliable results. This was because the flow rates being measured were not the true absolute potentials, and also because in the case of wells of high capacity the accurate measurement of the flow rates was impossible. Even in cases where the measured potential was relatively accurate, the results were not particularly useful in that they provided no information regarding the ability of a well to produce against various pipeline back pressures, nor any information regarding the delivery capacity of the well at declining reservoir pressures. In addition to this, the practice of flowing a well wide open often resulted in damage to the well by the coning of water into the well bore or by the "sloughing off" and production of sand particles from the formation. Often too, serious wastage of gas resulted. All things considered, the practice of testing a well "wide open" has been recognized as undesirable, and methods have been developed for assessing productive capacity by conducting well tests at reasonable and controlled rates of flow.

The Back Pressure Test

The basic work towards arriving at a practical test was carried out by the U. S. Bureau of Mines during the period 1929 to 1935. This work was documented in two Reports of Investigation by Pierce and Rawlins (66)(67) and culminated with the publication of the well-known and widely used Monograph 7 (72) of Rawlins and Schellhardt. The early work of Pierce and Rawlins actually first proposed the back pressure method of testing gas wells. Monograph 7 repeated the same basic material and equations and in addition included sections on testing equipment, gas measurement and tables which compiled the pressure differences from sandface to wellhead for various static and flowing conditions. These reports presented the empirical justification of the back pressure test, described the field conduct of the test, and analysed the results of such a test based on data obtained on 582 wells throughout the United States. Briefly, it was shown that if a gas well were appropriately tested by flowing it at several rates, a plot of the difference between the square of the static reservoir pressure and the square of the flowing bottom hole pressure versus the corresponding rate of flow should yield a straight line on log-log paper. It was then demonstrated how the plot could be employed to determine the well capacity at any flowing sandface pressure including absolute zero (corresponding to absolute open flow conditions), and how it could be used to predict the behavior of a well as the reservoir pressure declines.

Figure 1-1 presents a plot of the relationship proposed by Pierce and Rawlins for a typical Alberta gas well. The plot shows how the difference in pressures squared versus the corresponding flow rates for four test conditions result in a straight line relationship. This relationship was expressed by the following equation

$$Q = C\,(P_f^{\,2} - P_s^{\,2})^n \qquad (1\text{-}1)$$

where

Q = rate of flow, millions of cubic feet per day at standard conditions.

P_f = shut-in formation pressure, psia.

P_s = flowing sandface pressure, psia.

C = a coefficient which is determined mainly by the reservoir characteristics at the particular well.

n = an exponent which depends primarily upon the type of flow in the reservoir at the particular well. The value of 'n' is the reciprocal slope of the back pressure curve.

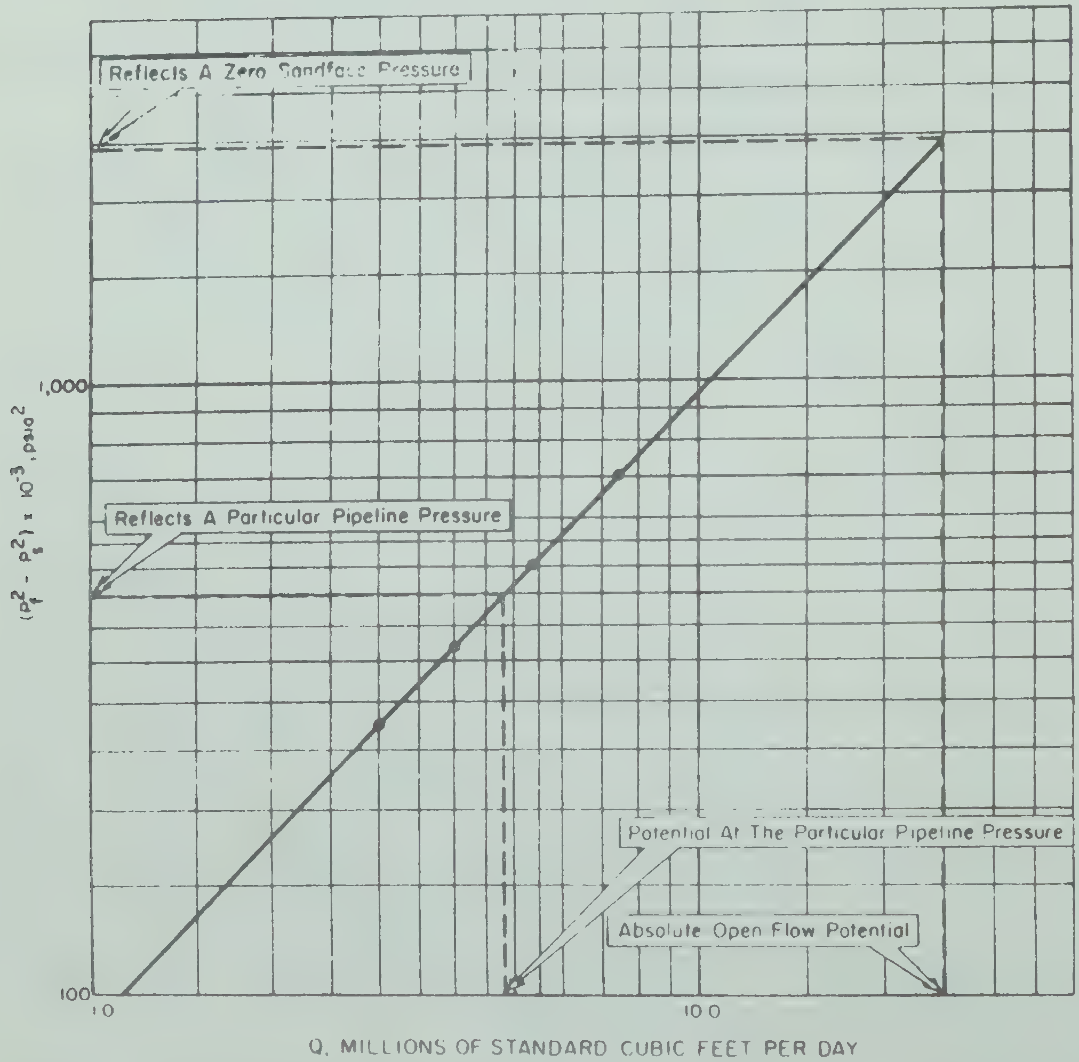

FIGURE I-1 — CONVENTIONAL BACK PRESSURE TEST PLOT

Figure 1-1 shows how the plot may be used to determine the absolute open flow potential and the capability of a well to flow against a particular pipeline pressure.

The back pressure test proposed by the U. S. Bureau of Mines was widely accepted and used by industry and regulatory authorities for a period of some 20 years. It was normal practice to produce wells below a maximum gas well allowable based on some constant percentage (usually 20 to 35 per cent) of the open flow potential. In 1954, the Alberta Oil and Gas Conservation Board recognized that the use of a fixed percentage of the open flow over the life of the pool is equivalent to a steady reduction in the sandface drawdown pressure over the entire pool life (97). Aware that an increasing safety factor over the pool life was unnecessary, the Board altered its policy and used the results of back pressure tests as a guide in setting allowables more nearly to reflect a uniform sandface drawdown at any reservoir pressure - i. e. allowables equivalent to an increasing percentage of the open flow potential as the reservoir pressure declines.

Pierce and Rawlins recognized that it is not always possible to measure the pressures directly down hole and presented a theory and method for converting wellhead pressure measurements to bottom hole or sandface conditions by taking into account the weight of the column of gas, and in the case of the flowing well, the pressure drop due to the friction. Their approach was based on observations at wells which were producing from relatively low pressure reservoirs with little or no liquid production. With the discovery of deep high-pressure and condensate type reservoirs, problems were encountered in determining sandface pressures from top hole measurements. Vitter (93) presented a method for calculating the sandface pressure for condensate wells.

The method combines the gas and liquid products of a separator in determining the weight of the gas column for the case where the condensate and the gas recovered from the separator exist in a single phase in the well bore. Subsequent to the publication of Vitter's paper, Shell Oil Company (105) pointed out that a slip had been made in the original paper with respect to the flowing case, and further modified the calculation. The "Modified Vitter's" method of calculating sandface pressures has received widespread acceptance and is in common use today.

More recently, a number of other investigators (24)(25)(68)(83) have proposed improved methods for calculating sandface pressures (particularly in the flowing case) in the single phase gas well. The best of the methods take advantage of recent developments (16)(29)(77)(78)(79) in the field of fluid mechanics in determining the friction resistance factor. In addition to this, much work has been done on the problem of determining the sandface pressure when both a gas and a liquid phase exist in the well bore (6)(11)(32)(33)(34)(37)(38)(54)(56)(69)(75)(87).

An improved understanding of the flow of fluids through porous media has made it clear that the empirical straight line relationship proposed by Pierce and Rawlins only results under conditions of stabilized and laminar flow in the reservoir, and in this case the reciprocal slope of the back pressure test curve 'n' should equal unity.

Work of Fancher, Lewis and Barnes (28) defined turbulent flow in porous media at an early date. It has since been shown (7)(27) that under conditions of high flow rates in the reservoir, where turbulence occurs, the back pressure "line" even for stabilized flow would curve upwards or reflect a value of 'n' decreasing from the value of unity expected at low flow rates. In most practical cases the variation in 'n' is not too great and it may be represented by a constant average

value at something less than unity. These considerations point out the danger of excessive extrapolation of the back pressure curve.

An even more important consideration in gas well testing is the problem of stabilization. It has been pointed out that the back pressure plot as proposed by Pierce and Rawlins will result in a straight line only if the flow rates and pressures are essentially stabilized. This condition is seldom attained, particularly in low permeability reservoirs where it is now recognized that the conventional back pressure test gives meaningless results, unless wells are flowed for prohibitive periods of time at each flow rate.

New Approaches to Testing Gas Wells

Cullender (23) has shown that a reasonably accurate back pressure curve may be obtained where the individual flow rates have not stabilized, provided, a certain testing procedure is adhered to. This procedure involves flowing the well at three or four flow rates for periods of equal duration, with each flow period beginning from comparable shut-in conditions. The shut-in conditions must be close enough to fully stabilized conditions so that any pressure build-up still occurring will not affect the recorded pressures during subsequent flow tests. (A rule of thumb useful in establishing whether a well pressure is sufficiently stabilized is that the pressure should be building up at a rate of less than about one-tenth of one per cent of the previously recorded pressure in 30 minutes). The resulting data points, when plotted in the conventional back pressure test manner, yield a straight line and the correct value of 'n'. An additional point is used from a run with an extended flow period approximating stabilized conditions.

A line drawn through this point and with the correct 'n', represents the true stabilized back pressure test line. The Cullender method, known as the "isochronal test", is consistent with the present day theory of gas well testing and is receiving increasing consideration from industry and regulatory bodies.

Another form of isochronal test has been presented by Katz et al (49). This "modified isochronal test" is receiving increasing support in industry. The essential modification is that each closed-in time between flow periods is also of the same duration as the flow period, and the new (unstabilized) closed-in pressure is used for calculating the difference of squares for the next flow point. While this procedure is not as readily justified on theoretical grounds as the true isochronal test, the results are very nearly the same.

In addition to these advances in the understanding, conduct and interpretation of flow tests, there has been increasing awareness of the importance of an accurate determination of the true static reservoir pressure. Many investigators (2)(3)(18)(26)(40)(43)(44)(52)(58)(59)(61)(62)(65)(89) have discussed the characteristics of pressure build-up in reservoirs based on the theory of fluid flow, and several methods have been developed for calculating the stabilized reservoir pressure from pressure-time data taken over a period of shut-in time substantially less than would be required to attain true stabilization. Out of an improved theoretical understanding of pressure build-up and drawdown data, has arisen the possibility of using such tests to supplement or even replace the results of back pressure tests.

Natural Gas Properties

In the early days, the only knowledge of gas properties needed

in conducting well tests were those required to calculate the flow rates, the gas gravity and compressibility factor. Moreover, they were needed only at the temperature and pressure of metering. With further development and refinement of the back pressure test, a knowledge of the effects of temperature and pressure on the compressibility factor has become important, and the viscosity of the gas gained significance. In the case of condensate wells, where the separator gas and liquid exists as a single phase in the well bore, the gravity, compressibility and viscosity of the combined well stream are important. These properties are best determined by laboratory measurement on a sample of the gas but they may also be calculated from the analysis of the gas.

A detailed discussion of the important properties of natural gas along with examples illustrating how they may be calculated or estimated is presented in Appendix A. Also included in the Appendix is a tabulation listing those physical properties of the constituent components of natural gas that are required in carrying out well test calculations.

Measurement of Flow Rates

The measurement of gas flow rates during testing operations is normally carried out by means of an orifice meter or a critical flow prover. In rare instances, an orifice well tester, a pitot tube or a choke nipple might be used.

When a well is tested directly into a pipeline, the gas measurement is normally carried out with an orifice meter. In addition to this, in many instances even though gas is being flared, a portable testing setup including an orifice type meter is used. The main advantages of this means of measurement are that it is one of the most accurate methods

available, and that it permits testing directly into a pipeline. Proper installation and maintenance, and an intelligent appraisal of the data gathered is critical in the use of an orifice meter. A number of publications are available which outline methods for calculating gas flow rates from basic orifice meter data and which include the necessary coefficients to correct these flow rates to certain temperature and pressure conditions. Among these are the American Gas Association Committee Report No. 3 (99), the California Natural Gasoline Association Bulletin No. TS-402 (102), and the back pressure testing manuals of the Interstate Oil Compact Commission (103) and the Kansas State Corporation Commission (104). The methods and coefficients published in these manuals, differ only slightly and in most cases have been based on work done by the American Gas Association. The Board accepts their Measurement Committee Report No. 3 as the standard reference for the installation and use of orifice meters.

The measurement of gas with the critical flow prover is based on the fundamentals of gas flow through an orifice under "critical conditions" or conditions where the velocity of flow has reached a maximum and remains constant. These conditions are normally assumed to occur, and the use of the critical flow prover is considered valid, when the ratio of the pressure downstream of the orifice to the upstream pressure is less than approximately 0.5. The main disadvantage in the use of the critical flow prover is that the produced gas must be blown to atmosphere. It is, however, a satisfactory means of measurement and its use involves a minimum of equipment. The U. S. Bureau of Mines Monograph 7 first published the method and provided appropriate coefficients for the use of the critical flow prover. However, the formula proposed in Monograph 7 was based on ideal gas behavior and it is now recognized that deviations from such behavior should be accounted for. Subsequently, the Texas

Railroad Commission and others have published appropriate modifications to the formula through use of the gas compressibility factor. Monograph 7 is still accepted as a standard in the design and field use of critical flow provers, but unless appropriate modifications are made, other publications such as the testing manuals of the Interstate Oil Compact Commission, the Kansas State Corporation Commission, or the Texas Railroad Commission (106) should be referred to for appropriate formulae and coefficients.

The pitot tube and orifice well tester are no longer used extensively in gas flow measurements. However, they do serve a useful purpose in gas measurement under certain conditions, particularly where the well produces from a low pressure reservoir. Choke nipples are sometimes used for measuring gas flow rates during testing operations when the gas must be flared to atmosphere and where the use of a critical flow prover or orifice meter is precluded by excessive pressure or other reasons.

Each of these lesser important methods of gas measurement is discussed in some detail in Monograph 7, however, it should be noted that the basic equations included therein do not incorporate the compressibility factor, and should be modified accordingly.

Measurement of Pressures

The accurate measurement of pressures corresponding to the previously discussed flow rates is also of great importance in gas well testing. Since interpretations of back pressure test results must be based on the theory of flow in the reservoir, it follows that the important pressure in interpreting the test is the reservoir pressure. Ideally, this pressure should be measured directly through use of an accurate

carefully calibrated bottom hole pressure gauge. There are many types of such gauges available today, all of which when used properly are quite adequate for obtaining accurate sandface pressures directly. A detailed discussion of the general working principles and use of common gauges is available in the Alberta Oil and Gas Conservation Board's Manual For The Conducting and Reporting of Bottom-hole Pressure Tests (98).

In some instances, due to mechanical difficulties or for other reasons, it is not practical to use a bottom hole gauge. In addition to this, for shallow wells the accuracy of conversion from the top hole to bottom hole pressures is acceptable, so bottom hole gauge measurements are unnecessary. In these situations, wellhead pressures are usually measured and converted to sandface conditions, as has already been mentioned. The highest possible accuracy in wellhead measurement is important, and for best results these pressures should be taken with a dead-weight gauge. The dead-weight gauge is an instrument which utilizes the balancing effect of an unknown pressure against a set of calibrated weights. A discussion of the operations of a dead-weight gauge is contained in the "Gas Measurement Manual" published by the American Gas Association (100).

There are many methods of conversion from wellhead to bottom hole pressures, each having advantages and disadvantages. While the method originally proposed by Pierce and Rawlins and detailed in Monograph 7, along with its equivalents in the form of the Texas Railroad Commission method, the Vitter's method and the Modified Vitter's method, is suitable under certain circumstances, other methods developed more recently are superior in most instances. A brief discussion of some of these methods is included in Appendix B, for the case of a single phase gas under shut-in and flowing conditions.

A less detailed discussion of the determination of bottom hole pressures from wellhead measurements is also included for the case where both gas and liquid phases exist in the well bore.

In the case where only the gas phase is present in the well bore, the Board concludes that for shallow, sweet gas wells several methods appear to give comparable results in terms of accuracy of bottom hole calculations. However, if a standard method for relatively correct pressure calculations regardless of well depth and type of gas is to be selected, the method of Cullender and Smith (24) is probably superior. This method has therefore been adopted by the Board for converting wellhead pressures to sandface conditions.

Experience indicates that regardless of the method used, calculations of sandface pressures at high flow rates (greater than 12-15 MMcfd) are often subject to excessive errors. The reason for this may be the difficulty in accurately estimating friction effects at very high flow velocities. For relatively deep, high capacity wells flowing at high rates, the sandface pressure drawdown is normally small whereas the losses due to friction are very high. Consequently, a small error in determining the weight of gas and the friction loss can render the calculated sandface pressure meaningless. For these reasons, in cases where wells will be tested at high flow rates, serious consideration should be given to directly measuring the sandface pressures wherever physically possible.

The Board also concludes that where two phase flow occurs in the well bore, the problems arising from the calculation of bottom hole pressures are virtually insoluble at this time. It is, therefore, recommended that wherever an appreciable amount of liquid phase is suspected in the well bore, the bottom hole pressure be directly measured.

Relation of Theory to Practical Well Tests

The discussion of the deficiencies in the conventional back pressure test, and the examination of the more advanced methods of testing, demonstrate the need of a thorough understanding of the theory of gas flow in the reservoir in order to obtain maximum utility from well tests. For these reasons discussion of the theory of both steady and unsteady state flow in the reservoir is presented in Appendix C. Included are reviews of the flow equations under both turbulent and laminar conditions, along with example problems to illustrate some practical uses of these equations.

2. THE FLOW TESTING OF GAS WELLS

As noted in part 1, Pierce and Rawlins were the first to propose and set out a method for testing gas wells by gauging the ability of the well to flow against various back pressures. This type of flow test has been designated the "conventional back pressure test" in this manual.

The conventional back pressure test involves the determination of the shut-in reservoir pressure. The well is then flowed at three or four different conditions, and the flow rate and the corresponding flowing sandface pressure is measured at each of these conditions. The results are plotted (as illustrated in Figure 1-1) as the difference in the squares of the formation and the flowing sandface pressures versus the corresponding flow rates.

This method of testing and the interpretation of the data is relatively simple, and the method has been considered the basic acceptable standard for testing gas wells for many years. For this reason, and because an understanding of some of the problems associated with the conventional back pressure test justifies and supports the development of more practical type flow tests, the theory of the conventional type test is reviewed.

Theory of the Conventional Back Pressure Test

At the outset it should be understood that the original development of the back pressure relationship by Pierce and Rawlins was based on empirical methods. In the period since development of the basic relationship, much has been learned regarding the flow of fluids in porous media.

This now permits a theoretical analysis and better understanding of, if not a complete justification for, the basic back pressure relationship.

The relationship is commonly expressed in the form of equation (1-1):

$$Q = C\,(P_f^2 - P_s^2)^n \tag{1-1}$$

where

Q = rate of flow, millions of cubic feet per day at standard conditions.

P_f = shut-in formation pressure, psia.

P_s = flowing sandface pressure, psia.

C = a coefficient which describes the positioning of the back pressure test curve.

n = an exponent which describes the slope of the back pressure test curve.

If we compare this equation to the radial laminar flow equation, described in Appendix C,

$$Q = \frac{0.7054 \times 10^{-6}\, kh\,(P_f^2 - P_s^2)}{\mu_a Z_a T_a \ln(r_f/r_s)} \tag{C-8}$$

where

Q = gas flow rate, millions of cubic feet per day at 14.65 psia and 60°F.

k = permeability, millidarcys.

h = thickness, feet.

P_f = formation pressure (at exterior drainage boundary), psia.

P_s = flowing sandface pressure, psia.

μ_a = average viscosity, cp.

Z_a = average compressibility factor.

T_a = average flowing temperature, ^oR.

r_f = exterior boundary radius, ft.

r_s = effective well bore radius, ft.

it is apparent that equations (1-1) and (C-8) are similar. In fact, if the coefficient 'C' is a constant, at a value of

$$C = \frac{0.7054 \times 10^{-6}\ kh}{\mu_a Z_\varepsilon T_a \ln(r_f/r_s)} \qquad (2\text{-}1)$$

and if 'n' is equal to unity, the equations are identical.

If the flow in a reservoir is steady-state and if the radius of drainage, r_f, is known to be a constant, the value of 'C' as described by equation (2-1) is in fact a constant for a particular reservoir pressure. Although flow in a reservoir is not a steady-state phenomenon, it has been determined that when the radius of drainage reaches the limits of a reservoir or when the drainage areas of two adjacent wells meet, "stabilized flow" occurs and although the flow is of an unsteady-state nature the steady-state equations approximate actual flow conditions. This matter is discussed in greater detail in Appendices C and D.

As a result, it is concluded that where flow rates and pressures represent stabilized conditions, the back pressure relationship as developed by Pierce and Rawlins, with 'n' equal to unity, is analogous to the steady-state laminar flow equation. Since flow in a reservoir is not normally completely laminar, one function of the exponent may be considered as a correction for the actual reservoir flow conditions. For completely stabilized flow, its value is largely a reflection of the type of flow (laminar, transitional, or turbulent) in the reservoir at a particular well.

One of the major problems related to the conventional back pressure test is the matter of stabilization. (This problem is discussed in detail

in Appendix D.) It has been pointed out that the back pressure test will result in the correct straight line relationship only if the values of the coefficient 'C' (equation (2-1)) reflected at each flow rate in a multi-point test are constant. Theoretically, this will occur if the effective radius of drainage, r_f, is the same for each flow rate. The easiest way to approximate this, is to flow the well at each rate to "stabilization". (The effective radius of drainage will then be the exterior boundary of the reservoir or the point at which the pressure gradients from two adjacent wells meet. In any case, it will be essentially the same for each flow point.)

In a reservoir of very high permeability, the time required to obtain stabilized flow rates and flowing pressures, as well as a stabilized shut-in formation pressure is usually not excessive. In this type of reservoir, a properly stabilized conventional back pressure test may be conducted in a reasonable period of time. On the other hand, in low permeability reservoirs the time required to even approximate stabilized flow conditions may be very long. In this situation, it is not practical to conduct a completely stabilized test, and since the results of an unstabilized test can be very misleading, the more recently developed methods of testing should be used to predict well behavior.

Isochronal Flow Test

The conventional back pressure test carried out under stabilized conditions, qualifies as an acceptable approach to attaining the straight line relationship which is essential in the proper interpretation of tests, because it extends each flow rate over a period of time sufficient to permit the radius of drainage to reach the outer edge of the reservoir or the point of interference between neighbouring wells. This ensures that the

radius of drainage, r_f, and the back pressure coefficient 'C' are constants, and except for the effects of turbulence, the back pressure relationship is a straight line representing stabilized flow conditions in the reservoir.

If each flow rate of a multi-point test extends for any other period of time insufficient for stabilization, but such that the effective radius of drainage is the same for each point, then the coefficient 'C' should also be a constant and the back pressure relationship a straight line.

Cullender (23) suggested that a series of flow tests at different rates for equal periods of time will result in a straight line relationship, and demonstrated that such a performance curve will have a value of the exponent 'n' essentially the same as that established under stabilized flow conditions. This method of testing is commonly referred to as the isochronal flow test and may be used in conjunction with only one stabilized flow point to replace a fully stabilized back pressure test. Briefly, the isochronal test consists of alternately closing in the well until a stabilized, or very nearly stabilized, pressure is reached and flowing the well at different rates for a set period of time. One flow test is conducted for a time period long enough to attain stabilized conditions. The observations reflecting the equal time flow periods are used to construct a back pressure line of the proper slope, and the final stabilized conditions are used to estimate the stabilized coefficient 'C' and thus the position of the curve.

The test proposed by Cullender, is based on the principle that the effective radius of drainage in a given reservoir is a function only of dimensionless time, and is independent of the flow rate. This assumption is discussed in Appendix E.

The true isochronal test requires that the pressure between each flow rate be allowed to return to the stabilized shut-in reservoir pressure. In very tight reservoirs, it is not always practical to attain a completely equalized reservoir pressure before the initial flow period, nor is it always practical during the test to close the reservoir in until the original pressure is regained. As a result, the true isochronal test proves impractical as a means of testing many wells.

Katz (49) has suggested that a modified isochronal test conducted with a closed-in period equal to the flow period may give satisfactory results provided the associated unstabilized closed-in pressure is used for P_f in calculating the difference of pressures squared for the next flow rate. This method has been used for testing many wells, and indeed has given results which appear quite satisfactory. The method, to be referred to as the "modified isochronal flow test", does not yield a true isochronal curve but closely approximates the true curve. This is illustrated in Appendix E.

More recently, several methods have been published for estimating the stabilized performance of gas wells on the basis of theoretical considerations and data obtained from short term flow tests. One such method is the "two flow method" as proposed by Carter, Miller and Riley.[14] This approach involves closing in a well until the pressure is equalized, flowing the well at a constant flow rate for a short period of time, again closing the well in until static conditions are attained, and finally flowing the well at a different and constant flow rate for a time equal to the first flow period. Calculations are then made using basic flow theory of the skin factor and the turbulence factor. These factors are then used to calculate the stabilized relationship between pressure drawdown and flow rates. This relationship can be plotted in the form of the conventional back pressure test curve. Winestock and Colpitts (96) have suggested a method for

interpreting the two flow rate form of test which does not require constant flow rates. This modification reduces greatly the expense and inconvenience of running such a test.

Methods such as the one described above are particularly useful when testing low permeability gas wells which are not connected to pipelines in that they estimate the stabilized performance of a well without wasting large amounts of gas. These more recent approaches to testing are being used more frequently by industry and undoubtedly will become more important in the future.

The modified isochronal test is probably the best all purpose multi-point flow test available for gauging a well's ability to produce gas. For this reason, it will be discussed in some detail. It is recognized that most of the matters discussed in connection with the modified isochronal test are of importance in the conduct and interpretation of any flow test, and is therefore understood that where applicable these matters will be given consideration regardless of the type of test being conducted. (Significant differences in the theory or conduct of other type tests will be referred to and compared with the modified isochronal test.)

Logically, the modified isochronal test (or the isochronal test) comprises three distinct parts. These are the establishment of the stabilized closed in reservoir pressure, the determination of the proper value of the exponent 'n' and the positioning of the back pressure curve to stabilized conditions. The discussion of the test will be handled therefore under three general headings.

Determination of Stabilized Reservoir Pressure

The stabilized formation pressure is normally determined by

closing in the well for an appropriate period of time prior to the initial flow period. In many reservoirs, due to the tight nature of the formation, it is not practical to close the well in until the stabilized pressure can be measured directly. In these instances, the well may be shut-in only for a period of time long enough to establish a high degree of pressure equalization in the area immediately surrounding the well bore. At the time when this initial closed-in formation pressure is no longer building up rapidly (say, less than 1/10 of one per cent of the previously recorded pressure in 30 minutes) the modified isochronal test method should give acceptable results with respect to the slope of the back pressure line. If the shut-in period is of sufficient length, and if the appropriate observations are made, the true static reservoir pressure (which is required for positioning the line), may be calculated from the pressure build-up data. Many methods have been developed for determining this static pressure from pressure-time data. Several of these methods are discussed in some detail in Part 4, "Interpretation of Build-up and Drawdown Data".

If the initial shut-in period is not long enough to permit either the measurement or calculation of the true static formation pressure (or if the necessary data have not been recorded), the pressure can probably be determined subsequent to the flow portion of the test with little or no effect on accuracy, provided, this is done within a reasonable period of time.

In the case of a new well in a reservoir or one which has produced only limited amounts of gas, the discovery pressure for the well may be used as the stabilized formation pressure.

Determination of Slope of Back Pressure Curve

The most commonly used method for determining the appropriate

slope of the back pressure relationship or the value of the exponent 'n' is to produce the well at a series of flow rates. In the case of either the modified or the true isochronal test, the flow periods are normally of equal duration, or at least observations are made at equal time intervals. (For a conventional back pressure test, each of the flow periods must be of sufficient length to ensure stabilized conditions, so theoretically they also should be of equal duration.)

Theoretically the flow periods for an isochronal test may be as short as a matter of minutes (10 minute flow periods have been plotted and give accurate results in some pools), but from a practical point of view should be long enough to obtain steady conditions with respect to the well bore, measurement facilities and other surface equipment. One hour flow periods are commonly used and in most reservoirs should give accurate results. For the modified isochronal test, the closed-in period between flow rates should be of approximately the same duration as the flow period, and in any case should be long enough that the pressure is no longer building up at a rate that will affect subsequent flow tests. The pressure immediately prior to each flow rate must be recorded and used as the formation pressure with respect to that rate. (For the isochronal test, the closed-in period is of sufficient duration to permit the pressure to return to the initial closed-in pressure, and for the conventional test, the closed-in period is held to a minimum.) The results of the isochronal test are plotted in the normal manner as the difference in the formation and the flowing pressures squared versus the corresponding flow rates. This results in a straight line relationship with a slope equal to the reciprocal of the exponent 'n'.

One of the problems that may arise in connection with back pressure testing (regardless of the method used), is the departure from

approximate laminar flow and the effects of the onset of turbulence as the gas flow rates are increased from point to point during a multi-point test. Binckley (7) has shown that for steady-state isothermal flow, the value of the exponent of the back pressure curve 'n' is 0.5 for completely turbulent flow. Since 'n' equals unity for completely laminar flow, the range of slopes to be expected for back pressure curves is 0.5 to 1.0.

The flow rates associated with virtually all back pressure tests are neither in the range of complete laminar nor complete turbulent flow. Thus, the degree of turbulence and its effect on 'n' will increase for each increasing flow rate. This effect will tend to reduce 'n' and if the range of rates of flow for a particular test is wide enough, the change in 'n' from near unity towards 0.5 will result in an upwards curvature of the back pressure line. Binckley (7), Elenbaas and Katz (27) and others have shown that this curvature does exist.

In most practical cases the curvature will be so slight that the back pressure test curve may be represented by a best fit straight line with a constant slope. However, inability to predict the degree of this curvature makes the interpolation of a back pressure curve to high flow rates very difficult. Govier (35) has presented a method for interpreting back pressure tests which attempts to account for the effect of turbulence. Whenever a test is to be extrapolated to flow rates well beyond those covered during the conduct of the test, it is recommended that consideration be given to correcting the test for the turbulence effect.

It follows that another approach to the determination of the appropriate slope of the back pressure curve (although one which is not part of any of the three standard type flow tests), might require only one accurate flow point. This point is then plotted as log $(P_f^2 - P_s^2)$ versus log Q, and a line reflecting laminar flow (slope of unity) is drawn through the point. The method of Govier (or any other appropriate

approach), is then used to adjust this line for turbulence at several different flow rates. Subsequently, a best fit straight line may be drawn through the points thus obtained. This approach is not feasible unless a considerable amount of information regarding certain characteristics of the formation can be determined with a reasonable degree of accuracy. Since this information must be obtained from build-up or draw-down tests, and since the method itself is based on the more recently developed theory, it is treated in more detail in Part 5, "The Estimation of Flow Behavior From Theory and Limited Data".

Positioning The Back Pressure Curve

For a properly conducted conventional back pressure test, the curve is automatically positioned to stabilized conditions because each of the flow rates is stabilized. However, for an isochronal type test, the final step involves positioning the curve to reflect stabilized conditions. Prerequisite to this step is a knowledge of the static formation pressure, the determination of which has been discussed earlier. Normally, the final flow rate of an isochronal test is extended until reasonably stabilized flow conditions are attained. The stabilized sandface pressure and the corresponding flow rate are then used to establish a stabilized point on the back pressure plot. A line through this point parallel to the isochronal line represents the stabilized back pressure relationship.

The main problem with respect to this method is establishing a suitable time for which the well should be flowed, or a time to approach a satisfactory degree of stabilization. It can be seen from Appendix D, that any established criterion is somewhat arbitrary, nevertheless, a standard is desirable.

If it is accepted that a suitable degree of stabilization will

have been attained when the radius of drainage has reached the outer edge of a one section spacing unit regardless of the actual density of development, several of the formulae presented and discussed in Appendix D can be used to calculate directly the time necessary to reach this condition. The Appendix shows that the time required to satisfy this criterion is surprisingly long. (The calculated time to stabilization may be several months for very tight reservoirs.) As a result, an attempt has been made to select an acceptable alternative and avoid having to position a back pressure curve on the basis of exceptionally long flow periods. Bearing in mind the use to be made of the flow test results, the Board concludes that a maximum time to stabilization of 15 days is satisfactory. In other words, if the calculated time for the pressure-radius profile to move to the edge of the one section spacing unit is longer than 15 days, the isochronal line may be positioned to reflect a "15 day stabilization", and this curve should be useful in deliverability and other related studies. Experience may be relied upon in extending or shortening the calculated stabilization time period, if previous tests in the same reservoir have shown that the defined condition of stabilization can be very closely approximated in some time other than that calculated.

The back pressure curve may also be positioned to represent stabilized conditions through calculation rather than through a 15 day flow test. Various methods (14)(19)(39)(70)(80)(84)(85) exist which permit the adjustment of the isochronal line to reflect the degree of stabilization corresponding to any length flow period, including that defined as adequately stabilized. They require a knowledge of certain reservoir characteristics normally determined from either a build-up or a drawdown test. Since the calculations are not considered part of the isochronal flow test, further details are included in Part 5.

In summary, the Board believes that a proper back pressure test should reflect a degree of stabilization which coincides with the lesser of the calculated time to stabilization (adjusted on the basis of experience) or 15 days, but recognizes that the adjustment to stabilization may be accomplished through either a lengthy flow test or a shorter flow test and associated calculations.

Frequency of Testing

With respect to the frequency of testing, it is clear that if the coefficient 'C' and the exponent 'n' of a back pressure relationship are constants over the life of a well, then only one back pressure test would be necessary to predict the behavior of the well over its life. Current knowledge of flow in porous media indicates that the radius of drainage, r_f, is a function of the time period over which a well has been flowing, and that the radius and thus the value of the coefficient 'C' becomes fixed when it has reached the outer edge of the reservoir or the areas to be drained by the well in question. (The flow has then become stabilized.) Even when this condition has been satisfied, a study of equation (2-1) leads to the conclusion that changes in the average gas compressibility and viscosity associated with the decline in pressure as the pool is depleted, will result in slight changes in the coefficient 'C'. At the same time, it may be assumed that the effect of the depletion of the pool on the exponent 'n' at a particular flow rate, should be only through the changing gas viscosity as the pressure declines. Since the variation in viscosity with pressure is very small, it follows that the slope of a back pressure curve should change very little throughout the producing life of a well. This fact, and the knowledge that in the absence of artificial stimulation, the coefficient 'C' changes very little from

year to year, supports the theoretical concept that only one back pressure test is required over the life of a well. Because of shortcomings in the theory and changes that may occur in reservoir properties, especially near the sandface, a complete test every five or six years would seem appropriate. The actual spacing of these tests will be influenced by many factors. Some of the most important ones, and their effect on a properly planned testing program are as follows:

(i) A gas well should be tested following any workover which may significantly change the ability of the well to deliver gas or if well performance indicates that an appreciable change has taken place in producing characteristics.

(ii) The number of wells drilled to and producing from the same pool will effect the testing program. Scheduling should be such that sufficient information is available (at least on an annual basis), to accurately trace the pressure history for the pool.

(iii) The possibility that damage may occur to a reservoir as a result of excessive producing rates also effects the timing of flow tests. If for a particular reservoir, strict control of maximum producing rates is necessary to prevent well damage, the frequency of testing must be increased to ensure that adequate information regarding the behavior of the wells is available at all times.

(iv) The ultimate market to which the gas will be delivered is an important factor. If the market being served requires current and highly accurate information

regarding the deliverability of the well, then a greater number of tests are necessary.

Acceptance of the premise that the exponent 'n' will remain essentially constant over the life of the well leads to the use of the so called "single point test". This procedure involves completion of a multi-point stabilized test to determine the exponent 'n' and the stabilized coefficient 'C'. Thereafter, if there is reason to believe that 'C' may have changed, only one stabilized flow point is obtained and the new back pressure curve is constructed by drawing a line with a reciprocal slope equal to the known 'n', but through the newly established point. The most desirable testing program, planned to yield the maximum amount of technical information in the most economical manner, should include a series of both multi-point and single point tests, spaced effectively over the life of a well.

3. FIELD CONDUCT AND REPORTING OF DATA FOR BACK PRESSURE TESTS

Earlier sections and corresponding appendices have indicated the Board's views with respect to the type of back pressure test which is recommended, the frequency of testing, and the suggested method for converting surface recorded pressures to sub-surface pressures. This section is intended as an appropriate conclusion regarding each of these matters. The discussion of field procedure will deal primarily with the conduct of a modified isochronal test, however, mention will be made where the procedure must be varied for other types of test. In a similar manner, the discussion of the calculations related to the test assumes use of the Cullender and Smith method (24) for converting pressures from top hole to bottom hole conditions. The standard forms presented for the recording of field notes and the calculating of results have been prepared specifically for use with these methods, although the form for recording field notes may be used for any type test. This is not true with respect to the calculation sheet.

Complete results of a typical modified isochronal test are shown on Figures 3-1 to 3-4 inclusive, to facilitate an understanding of the forms.

Field Conduct of the Back Pressure Test

The conventional back pressure test is satisfactory only in reservoirs where stabilized conditions can be attained in a relatively short period of time. Where more practical, a form of the isochronal test should be conducted.

In prescribing field conduct of the test, considerable use has been made of the recommended form of rules of procedure included in the

Interstate Oil Compact Commission back pressure test manual (103). Details with respect to proper field conduct are listed below in point form. Reference is made to Figure 3-1, a suggested form for the recording of field notes.

A. Shut-in Pressure

1. The well to be tested shall be produced prior to the initial shut-in period for a time sufficient to clear the well bore of any liquid hydrocarbons or water that may have accumulated. In the case where the well is tied into a pipeline the flow rate shall be large enough to clear the well bore of liquids. If the well cannot be cleared of liquids while producing into a pipeline, it shall be blown to atmosphere to remove the liquids.

Where the well is not connected to a pipeline, it shall be blown to atmosphere until it is obvious the well bore is cleared of liquids. Particulars of blowing the well shall be recorded in detail as shown in Figure 3-1.

2. The well shall be shut-in for a period long enough to ensure stable pressure conditions. If the static pressure is to be determined from this portion of the test, the period must be long enough to permit the gathering of sufficient pressure-time data to calculate the static pressure by one of the methods presented in Part 4 or any other acceptable procedure and in all likelihood need not be in excess of six days. (It may be recalled from the previous section that the modified isochronal test gives satisfactory results even when the initial shut-in pressure is not completely stabilized, provided the rate of pressure build-up is very small when the flow test begins. A rule of thumb is that the rate of build-up should be less than about 1/10 of one per cent of the previously recorded pressure in 30 minutes.)

If there is reason to believe that an appreciable accumulation of liquids may occur in the well bore during the shut-in period, the sandface pressures shall be directly measured with a bottom hole pressure gauge.

B. Flow Periods

1. The well shall be produced for a series of four different flow rates. If four significantly different flow rates cannot be obtained, three may suffice. (In the case of a conventional test, each point must be stabilized so theoretically should be of equal duration. For an isochronal type of test, the flow periods need not be of equal duration but at least one observation must be taken at the same time interval of each flow period. It therefore is normally convenient to make the flow periods of equal duration for all but the long or stabilized flow point.)

2. The lowest flow rate shall be a rate sufficient to keep the well bore clear of liquids. In some instances this may not be feasible if a sufficient spread in points is to be obtained, however, no appreciable quantity of liquids should be permitted to accumulate in the well bore during testing at the lowest flow rate.

3. Care must be taken to ensure an adequate spread of the flow points. (A widely used rule is that the drawdown for the lowest flow rate be at least five per cent of the static wellhead pressure, and that the drawdown for the highest flow rate be no greater than some twenty-five per cent of the static pressure.) Care must be taken at all times, to prevent well damage by unnecessarily coning water into the well bore at the higher drawdown rates.

4. An increasing flow rate sequence shall be employed. (If the sequence is reversed because it is impossible to obtain the required

number of flow rates without accumulating liquids in the well bore at the lower rates, then the modified isochronal test method loses accuracy, and may not be acceptable.)

5. For an isochronal type test, pressure observations shall be taken during each of the flow periods at equal time intervals. Each flow period must be long enough to ensure steady state conditions with respect to measurement and other surface facilities. In no case, however, shall they be less than one hour. One flow period shall be of sufficient duration to satisfy the accepted criterion of stabilized flow, or be of sufficient length that the appropriate observations may be made to permit a calculation to reflect stabilized conditions. By taking appropriate observations at time intervals consistent with observations made during other flow periods, the extended flow rate may serve as a point in establishing both the slope and position of the line.

For a conventional back pressure test, each flow rate shall satisfy the established criterion of adequate stabilization.

6. For a modified isochronal test, the closed-in time between each flow period should be of approximately the same duration as the flow period and in any case shall be long enough that the pressure is no longer building up at a rate that will appreciably affect subsequent flow tests.

For an isochronal test the shut-in time shall be sufficient to permit the pressure to return to a pressure comparable to that prior to the initial flow period.

For a conventional test, the shut-in time shall be held to a minimum.

7. In cases of deep wells with high flow rates, and particularly where two phase flow is suspected, wherever possible the flowing and closed-in sandface pressures should be directly measured with a bottom

hole gauge. In other cases, the flowing and closed-in pressures shall be measured with a dead-weight gauge and recorded on Figure 3-1, at least twice during each flow and closed-in period. One of the recorded pressures shall be at the end of each period.

8. If bottom hole pressures are to be calculated, the gas temperature at the wellhead shall be recorded during the closed-in and each flow period. To facilitate this a temperature well should be installed in the wellhead.

9. The gas flow rates during each of the flow periods shall be measured by an orifice meter, critical flow prover, orifice well tester, or other suitable metering device in good operating condition. The measurement, and subsequent calculation of flow rates, shall be in accordance with the procedures set out in the publications referred to in Part 1 (72)(99)(100)(101)(102)(103)(104)(106).

10. Where wellhead separation takes place, in addition to measurement of separator gas as described in item 9, the separator liquid producing rate shall be accurately measured and recorded along with the gas flow data on Figure 3-1. The measurement of the produced liquids shall be such that it permits the accurate determination of the producing gas-liquid ratio for each flow rate.

11. Where the bottom hole pressures are to be calculated, the specific gravity of the produced gas, as well as liquids where separation takes place, shall be accurately determined and recorded. This may be done by direct measurement using proper techniques, or alternately by careful sampling, analysing and the necessary computations. The determination of gravity need be carried out during one flow period only, preferably the second or third.

The Calculating, Plotting and Reporting of Test Results

This discussion assumes that the Cullender and Smith method (see Appendix B) of calculating sandface pressures from wellhead observations will be employed. As a result, the suggested form of calculation sheet (Figure 3-2) to which the discussion refers, is designed specifically for use with this particular method and is not applicable to other methods. Since each of the methods of calculating sandface pressures from wellhead measurements bear some similarity to each other, the rules set out as standard for use with the Cullender and Smith method should be followed where applicable when other methods of calculation are employed.

Figure 3-2 serves for converting wellhead observations to sandface pressures, so that it is unnecessary to complete this form when bottom hole pressures are measured directly.

Once again, the specific instructions for calculating the sandface pressures are presented in point form.

1. The effective diameter, d, of the flow string is the internal diameter of the string through which flow is taking place. In the case of annular flow an equivalent diameter is calculated from equation (B-31).

2. The length of the flow string, L, is the distance from the wellhead to the mid-point of the producing perforations.

3. In cases where the total well effluent is separated into a gas and a liquid at the wellhead, but existed as a gas in the well bore, the gravity of the recombined effluent is determined. This may be done through use of equation (A-4) and Figure A-9, as illustrated in Example A-3, all in Appendix A. If an analysis of the recombined well effluent is available, the gravity may be determined from the molecular

weight of the mixture, as illustrated in Example A-1.

4. The pseudo critical properties of the well effluent are determined from the recombined analysis by calculation as in Example A-1; or from the gravity of the effluent using Figure A-1.

5. The formation temperature, T_s, is based on measured temperatures in the same reservoir wherever possible. If no such measurements exist, the formation temperature may be estimated on the basis of similar reservoirs in the immediate proximity, or on the gradients suggested in Figure 3-5.

6. For the static case, the wellhead temperature, T_w, is taken as the mean annual surface temperature if the closed-in period has been extensive (greater than 24 hours). If the closed-in period has been short, such as will be the case in a modified isochronal test, it is probably more accurate to estimate a wellhead gas temperature closer to the final flowing gas temperature immediately prior to shut-in. Figure 3-6 presents mean surface temperatures for the Province of Alberta.

In the flowing case, the wellhead temperature is measured directly.

7. The gas flow rate calculations are carried out in an approved manner, depending upon the method of measurement. Where liquids have been produced at a separator, they are converted to an equivalent quantity of gas. This may be accomplished by using the vaporizing volume ratios as presented in Figure A-9. The gas-equivalent of the produced liquids is then added to the separator gas to get the total gas production. It is upon this total that the friction determination is based.

8. For the static case, and in the absence of a known temperature profile, the average temperature, T_a, is taken as the arithmetic average of the mean annual surface temperature and the formation temperature.

In the flowing case, the average temperature is estimated as the average of the recorded flowing wellhead temperature and the formation temperature.

9. The determination of the compressibility factor, Z, is carried out by one of the methods illustrated in Examples A-1 and A-2.

10. For purposes of estimating the viscosity (for friction factor determinations), an average well bore pressure and temperature (for all flow rates) may be estimated from a knowledge of the wellhead pressures and temperatures, the formation temperature, and an estimated formation pressure. The viscosity of the gas at these average conditions can be determined from Figures A-10 to A-12, as illustrated in Example A-4. This average viscosity may be used in the determination of the friction factor for all flow rates without introducing appreciable errors.

11. The relative roughness, δ/d, of the flow pipe is determined from Figure B-2. In the absence of special knowledge of the pipe condition, the absolute roughness, δ, may be taken as 0.0006.

12. The Reynolds Number, N_{Re}, for each flow rate is calculated from equation (B-18).

13. The friction factor, f, for each flow rate is determined from Figure B-1.

14. The friction effects as included in the Cullender and Smith equation, F^2, are determined for each flow rate from equation (B-22).

15. The values of I_w, I_a and I_s are calculated from equation (B-5) in the static case and (B-27) in the flowing case.

16. The trial and error calculations of P_a and P_s (from equations (B-8) and (B-9) in the static case and (B-29) and (B-30) in the flowing case), are repeated until the calculated pressure values are within one psi of the assumed values. These calculations are illustrated in Examples B-1 and B-2.

17. In both the static and flowing cases, the final pressures are corrected using Simpson's rule, equations (B-10) or (B-28).

The results of the back pressure test shall be recorded on Figure 3-3, the summary sheet. The producing rates and the pressure data (where sandface pressures are calculated), are recorded under item A. If the sandface pressures have been measured directly, they are recorded under item B. The shut-in pressures are recorded under item C. (In the case of a conventional back pressure test, only one shut-in pressure will be recorded.) The plotting data, item D, are made up of the appropriate shut-in and flowing pressures and the total gas flow rates. These results are plotted as Q versus $(P_f^2 - P_s^2)$, as illustrated in Figure 3-4. The plot should be made on equal scale log-log paper, and a straight line drawn through a minimum of three points. If a straight line is not indicated by at least three points, consideration should be given to retesting the well. The reciprocal slope of the line is the exponent 'n'. If the value of 'n' is greater than 1.0 or less than 0.5, consideration should be given to retesting the well, unless experience with wells in that pool indicates that a different 'n' value would not be obtained.

If a well has been retested, and the test is still unsatisfactory, the best fit line may be drawn through the points of the test which appears to be the most acceptable. If the resulting 'n' is greater than 1.0, a line reflecting an 'n' of 1.0 shall be drawn through the highest flow rate point. If the 'n' is less than 0.5, a line reflecting an 'n' of 0.5 shall be drawn through the lowest flow rate point. In any case the relationship resulting from this second unsatisfactory test is only an estimated one, and consideration should be given to a retest within a one year period. The retest should involve alterations in test procedure in an attempt to obtain a satisfactory relationship. This change may involve direct sandface pressure measurements if two-phase flow appears to be a possibility,

or may lead to another type of flow test.

In the case of an isochronal test, the line must be positioned to reflect stabilized conditions. This is done by plotting the stabilized value of $(P_f^2 - P_s^2)$ versus the appropriate flow rate. A line of reciprocal slope 'n' is drawn through the point, as is illustrated in Figure 3-4. The data corresponding to this flow point may conveniently be designated in the field notes, or on the calculation sheet and the summary sheet, as run number 4(a) for a four point test.

FIGURE 3-1

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BACK PRESSURE TEST - FIELD NOTES - PAGE 1

Well ABC Gas Company No.1 Location Lsd.-Twp.-Rge.-Mer. Date August 1, 1963
Field Gas Field Pool Gas Pool Tested By J.L.Smith
Casing Size 5 1/2 Weight 15.5 Grade J-55 Producing Interval 6732-6760
Tubing Size 2 Weight 4.7 Grade J-55 Producing Through Tubing
Well Blown 10 minutes. Description of Blow Water-Condensate Spray, Clear in 5 min.

Gas Gravity 0.691 Source Analysis-2nd Run Mean Annual Surface Temp. 37 °F
Liquid Gravity 0.748 Source Analysis-2nd Run B.H. Temp. 163 °F Source Temp.Bomb
Date Shut-In July 25 Time 8:00 am Total Time Shut-In 168 hrs.

SHUT-IN PERIOD NO. 1 (Initial)

Time of Observation			Observed Pressures				Wellhead Temp.-°F	Remarks
			Tubing		Casing			
Date	Time	Shut-In Time Hrs.	psig	psia	psig	psia		
July 25	8:00am	0	1820	1834			60	Casing Pressures Not
July 28	8:00am	72	1912	1926			53	Measured - Packer in
July 31	8:00am	144	1948	1962			54	Casing
July 31	12:00Noon	148	1948	1962			51	
Aug.1	8:00am	168	1948	1962			62	

FLOW PERIOD NO. 1

Duration of Run 1 hrs. Orifice Size 2.000 Meter Run or Prover Size 4.026
High Pressure Separator - Operating Pressure 890 psig Temp. 90 °F
Low Pressure Separator - Operating Pressure ______ psig Temp. ______ °F

Date	Time	Flow Time Hours	Wellhead Readings			Meter or Prover Data			Condensate Prod-Bbls	Water Prod-Bbls
			Tubing psig	Casing psig	Temp. °F	Pf psia	Hw	Temp. °F		
Aug.1	8:00am	0	1933		60	900	37	86		
	8:30am	0.50	1881		73	900	37	86		
	8:45am	0.75	1865		79	900	37	88		
	9:00am	1.00	1853		82	900	37	[illegible]	Total 1.00	Total 0.25

SHUT-IN PERIOD NO. 2 (Intermediate-for use with isochronal type tests)

Date	Time	Shut-In Time Hrs.	Tubing Pressure		Casing Pressure		Wellhead Temp. °F
			psig	psia	psig	psia	
Aug.1	9:00am	0	1853	1867			82
	9:30am	0.50	1930	1944			77
	9:45am	0.75	1946	1960			73
	10:00am	1.00	1954	1968			71

FLOW PERIOD NO. 2

Duration of Run 1 hrs. Orifice Size 2.750 Meter Run or Prover Size 4.026
High Pressure Separator - Operating Pressure 945 psig Temp. 92 °F
Low Pressure Separator - Operating Pressure ______ psig Temp. ______ °F

Date	Time	Flow Time Hours	Wellhead Readings			Meter or Prover Data			Condensate Prod-Bbls	Water Prod-Bbls
			Tubing psig	Casing psig	Temp. °F	Pf psia	Hw	Temp. °F		
Aug.1	10:00am	0	1954		71	955	29	89		
	10:30am	0.50	1786		94	955	29	90		
	10:45am	0.75	1741		102	955	29	90		
	11:00am	1.00	1713		110	955	29	90	Total 1.85	Total 0.46

OIL AND GAS CONSERVATION BOARD
BACK PRESSURE TEST - FIELD NOTES - PAGE 2

SHUT-IN PERIOD NO. 3 (Intermediate - for use with isochronal type tests)

Date	Time	Shut-In Time Hrs.	Tubing Pressure		Casing Pressure		Wellhead Temp. °F
			psig	psia	psig	psia	
Aug. 1	11:00am	0	1713	1727			110
	11:30am	0.50	1898	1912			103
	11:45am	0.75	1937	1951			94
	12:00Noon	1.00	1959	1973			87

FLOW PERIOD NO. 3

Duration of Run 1 hrs. Orifice Size [illegible] Meter Run or Prover Size 4.026
High Pressure Separator - Operating Pressure 940 psig Temp. 106 °F
Low Pressure Separator - Operating Pressure ______ psig Temp. ______ °F

Date	Time	Flow Time Hours	Wellhead Readings			Meter or Prover Data			Condensate Prod-Bbls	Water Prod-Bbls
			Tubing psig	Casing psig	Temp. °F	Pf psia	Hw	Temp. °F		
Aug. 1	12:00Noon		1959		87	953	42	104		
	12:30pm	0.50	1613		112	953	42	105		
	12:45pm	0.75	1541		117	953	42	105		
	1:00pm	1.00	1489		120	953	42	105	Total 2.80	Total 0.[illegible]

SHUT-IN PERIOD NO. 4 (Intermediate - for use with isochronal type tests)

Date	Time	Shut-In Time Hrs.	Tubing Pressure		Casing Pressure		Wellhead Temp. °F
			psig	psia	psig	psia	
Aug. 1	1:00pm	0	1489	1503			120
	1:30pm	0.50	[illegible]	[illegible]			109
	1:45pm	0.75	1924	1938			106
	2:00pm	1.00	1960	1974			104

FLOW PERIOD NO. 4

Duration of Run 1 hrs. Orifice Size 3.000 Meter Run or Prover Size 4.02[illegible]
High Pressure Separator - Operating Pressure 915 psig Temp. 116 °F
Low Pressure Separator - Operating Pressure ______ psig Temp. ______ °F

Date	Time	Flow Time Hours	Wellhead Readings			Meter or Prover Data			Condensate Prod-Bbls	Water Prod-Bbls
			Tubing psig	Casing psig	Temp. °F	Pf psia	Hw	Temp. °F		
Aug. 1	2:00pm	0	1960		104	923	70	112		
	2:30pm	0.50	1346		118	923	70	114		
	2:45pm	0.75	1281		123	923	70	114		
	3:00pm	1.00	1219		125	923	70	114	Total 3.50	Total 0.83

Remarks: ______

OIL AND GAS CONSERVATION BOARD

BACK PRESSURE TEST - FIELD NOTES - PAGE 3

SHUT-IN PERIOD NO. 5 (Intermediate - for use with isochronal type tests)

Date	Time	Shut-In Time Hrs.	Tubing Pressure psig	Tubing Pressure psia	Casing Pressure psig	Casing Pressure psia	Wellhead Temp. °F
Aug.1	3:00pm	0	1219	1233			125
	3:30pm	0.50	1800	1814			112
	3:45pm	0.75	1910	1924			108
	4:00pm	1.00	1955	1969			105

FLOW PERIOD NO. 4(a)

Duration of Run 144 hrs. Orifice Size 2.000 Meter Run or Prover Size 4.026
High Pressure Separator - Operating Pressure 890 psig Temp. 90 °F
Low Pressure Separator - Operating Pressure ______ psig Temp. ______ °F

Date	Time	Flow Time Hours	Wellhead Readings: Tubing psig	Wellhead Readings: Casing psig	Wellhead Readings: Temp. °F	Meter or Prover Data: Pf psia	Meter or Prover Data: Hw	Meter or Prover Data: Temp. °F	Condensate Prod-Bbls	Water Prod-Bbls
Aug.1	4:00pm	0	1955		105	900	37	[illegible]		
Aug.2	4:00pm	24	1860		102	900	37	[illegible]		
Aug.3	4:00pm	48	1843		100	900	37	[illegible]		
Aug.4	4:00pm	72	1835		100	900	37			
Aug.5	4:00pm	96	1831		101	900	37			
Aug.6	4:00pm	120	1830		101	900	37	[illegible]		
Aug.7	4:00pm	144	1829		100	900	37	[illegible]	Total 138.5	Total [illegible]

SHUT-IN PERIOD NO. (Intermediate - for use with isochronal type tests)

Date	Time	Shut-In Time Hrs.	Tubing Pressure psig	Tubing Pressure psia	Casing Pressure psig	Casing Pressure psia	Wellhead Temp. °F

FLOW PERIOD NO.

Duration of Run ____ hrs. Orifice Size ______ Meter Run or Prover Size ______
High Pressure Separator - Operating Pressure ______ psig Temp. ______ °F
Low Pressure Separator - Operating Pressure ______ psig Temp. ______ °F

Date	Time	Flow Time Hours	Wellhead Readings: Tubing psig	Wellhead Readings: Casing psig	Wellhead Readings: Temp. °F	Meter or Prover Data: Pf psia	Meter or Prover Data: hw	Meter or Prover Data: Temp. °F	Condensate Prod-Bbls	Water Prod-Bbls

Remarks:

FIGURE 3-2

OIL AND GAS CONSERVATION BOARD

BACK PRESSURE TEST - CALCULATION SHEET - PAGE 1.

Well ABC Gas Company No.1 Location Lsd.-Twp.-Rge.-Mer. Field Gas Field

Pool Gas Pool Tested by J. L. Smith Calculated by J.T.Doe

Csg. Size 5 1/2 Wt. 15.5 Grade J-55 Producing Through Tubing Length of Flow String 6,746

Tbg. Size 3 Wt. 9.3 Grade J-55 Effective Diameter of Annulus ______

Perforations 6732-6760 Gas Gravity 0.691 Liquid Gravity 0.748

Gas-Liquid Ratio, Rc 158,000. Properties of Mixture - Gravity 0.710 Pc 671 Tc 378

Mol. % - CO_2 2.04 H_2S 0 N_2 2.47 Source Analysis-Sample from 2nd flow

Mean Annual Surface Temp. 37 °F. Reservoir Temp. 163 °F Source Temp.Bomb

STATIC CASE

$$I = TZ/P$$

$0.0375G\frac{L}{2}$	P_w psia	T_w °R	Z_w	I_w	P_a psia	T_a °R	Z_a	I_a	P_s psia	T_s °R	Z_s	I_s	P_s psia Corr.
89806	1962	522	0.695	0.1849	2193	572.5	0.761	0.20[illegible]	2406	623	0.837	0.2167	2405
89806	1968	531	0.714	0.1926	2193	577.0	0.7[illegible]	0.206[illegible]	2405	623	0.837	0.2168	2404
89806	1973	547	0.744	0.2063	2187	585.0	0.797	0.2132	2395	623	0.837	0.2177	2395
89806	1974	564	0.773	0.2209	2175	593.5	0.807	0.2199	2383	623	0.837	0.2188	238[illegible]
89806	1969	565	0.775	0.2224	2172	594.0	0.807	0.2207	2376	623	0.837	0.2195	237[illegible]

FLOWING CASE Viscosity, μ 0.0164 Absolute Roughness, δ 0.0006 Relative Roughness, δ/d [illegible]

$$I = P/TZ \Big/ \left[F^2 + \frac{(P/TZ)^2}{1000}\right]$$

Flow No.	Q Total MMcfd	*Reynolds Number $N_{Re} = \frac{20,011\ G\ Q}{\mu d}$	Friction Factor f-(Figure B-1)	*Friction Effects $F^2 = \frac{2.6665\ f\ Q^2}{d^5}$	P_w psia	T_w °R	Z_w
1	4.[illegible]	1.37×10^6	0.0036	0.00088	18[illegible]	542	0.74[illegible]
2	[illegible]	2.59×10^6	0.0035	0.00308	1727	570	[illegible]
3	13.[illegible]	3.82×10^6	0.0034	0.00653	1503	560	[illegible]
4	16.[illegible]	4.77×10^6	0.0034	0.01017	1233	585	[illegible]
4(a)	4.[illegible]	1.37×10^6	0.0036	0.00088	1843	560	0.77[illegible]

FLOWING CASE (Continued)

Flow No.	$37.5\ G\ \frac{L}{2}$	I_w	P_a psia	T_a °R	Z_a	I_a	P_s psia	T_s °R	Z_s	I_s	P_s - psia Corrected
1	[illegible]	[illegible]	2081	582	0.796	213	2289	623	0.838	218	2289
2	[illegible]	217	1937	596	0.819	211	2152	623	0.840	206	2152
3	[illegible]	192	1740	602	0.835	187	1983	623	0.845	182	1983
4	89806	152	1525	604	0.849	156	1813	623	0.852	156	1811
4(a)	89806	224	2043	582	0.81[illegible]	224	2244	623	0.839	222	224[illegible]

*For annular flow refer to equations B-18a, B-14a, B-23a & B-24a.

Remarks: ______

FIGURE 3-2

OIL AND GAS CONSERVATION BOARD

BACK PRESSURE TEST - CALCULATION SHEET - PAGE 2.

CASE 1. FLOW THROUGH PROVER

$$Q = C\ P\ F_g\ F_t\ F_{pv}$$

Run No.	C	P	F_g	F_t	F_{pv}	Q Mcfd

CASE 2. FLOW THROUGH METER RUN

$$Q = 24\ C'\ H_w P_f$$

$$C' = F_b\ F_g\ F_{tf}\ F_{pv}\ F_{pb}\ F_{tb}\ F_r\ Y\ F_m$$

Run No.	F_b	F_g	F_{tf}	F_{pv}	F_{pb}	F_{tb}
1	842.12	1.2030	0.9741	1.083	1.0055	1.0000
2	1746.7	1.2030	0.9723	1.087	1.0055	1.0000
3	2194.9	1.2030	0.9594	1.077	1.0055	1.0000
4	2194.9	1.2030	0.9518	1.066	1.0055	1.0000
4(a)	842.12	1.2030	0.9741	1.083	1.0055	1.0000

FLOW THROUGH METER RUN (Continued)

Run No.	F_r	Y	F_m	C'	$\sqrt{H_w P_f}$	Q Mcfd
1	1.0002	1.0002	-	1,075.0	182.5	[illegible]
2	1.0004	1.0001	-	2,234.1	[illegible]	[illegible]
3	1.0003	1.0002	-	2,744.7	200.1	[illegible]
4	1.0003	1.0003	-	2,695.4	254.2	[illegible]
4(a)	1.0002	1.0002	-	1,075.0	182.5	[illegible]

FIGURE 3-3

OIL AND GAS CONSERVATION BOARD

BACK PRESSURE TEST - SUMMARY SHEET

Well ABC Gas Company No.1 Location Lsd.-Twp.-Rge.-Mer. Field Gas Field

Pool Gas Pool Tested by J.L.Smith Calculated by J.T.Doe

Casing Size ________ I.D. Tubing Size 2.992 I.D. ______ O.D. ____ Produced Through Tubing

Perforations 6732-6760 Length of Flow String 6746

Properties of Well Effluent-Sp. Gravity 0.710 Pc 671 Tc 376

A. FLOW TEST DATA

Run No.	Tubing psia	Casing psia	Calculated Sandface Press. psia	Gas Flow Rate MMcfd	Condensate Prod. Bbl/d	Total Flow Rate MMcfd	Water Prod. Bbl/d
1	1867		2289	4.718	24.0	4.728	5
2	1727		2152	8.938	44.4	8.957	11
3	1503		1983	13.205	67.2	13.233	17
4	1233		1811	16.475	84.0	16.510	20
4(a)	1843		2245	4.717	23.1	4.727	5

B. BOMB MEASURED PRESSURES Datum ____________

Flow Rate No.	Date	Time	Depth	Corr. Press. Run Depth - psia	Corr. Press. @ Datum - psia	Remarks

C. SHUT-IN DATA (Note: For regular back pressure tests only one shut-in required.)

Shut-In No.	Pressure - psia		Shut-In Time-Hrs.	Shut-In No.	Pressure - psia		Shut-In Time-Hrs.
	Wellhead	Sandface			Wellhead	Sandface	
1 (Initial)	[illegible]	2405	168	4	1974	2382	1
2	[illegible]	2404	1	5	1969	2376	1
3	1973	2395	1				

D. PLOTTING DATA

P_f psia	$P_f^2 \times 10^{-3}$ psia^2	P_s psia	$P_s^2 \times 10^{-3}$ psia^2	$P_f^2 - P_s^2 \times 10^{-3}$ psia^2	Q Total MMcfd
2405	5784	2289	5240	544	4.728
2404	5779	2152	4631	1148	8.957
2395	5736	1983	3932	1804	13.233
2382	5674	1811	3280	2394	16.510
2405	5784	2245	5040	744	4.727

A.O.F. 26.6 MMcfd, n 0.845, C 5.14×10^{-5} : Note: $C = \frac{Q}{(P_f^2 - P_s^2)^n}$

Total volume of gas produced during test 30.2 MMcf.

Remarks: The figures under Gas Flow Rate, in Part A, represent metered gas plus gas flashed from the stock tank at 0.364 Mscf/STBbl.
The vapour volume of condensate is 0.420 Mscf/STBbl.

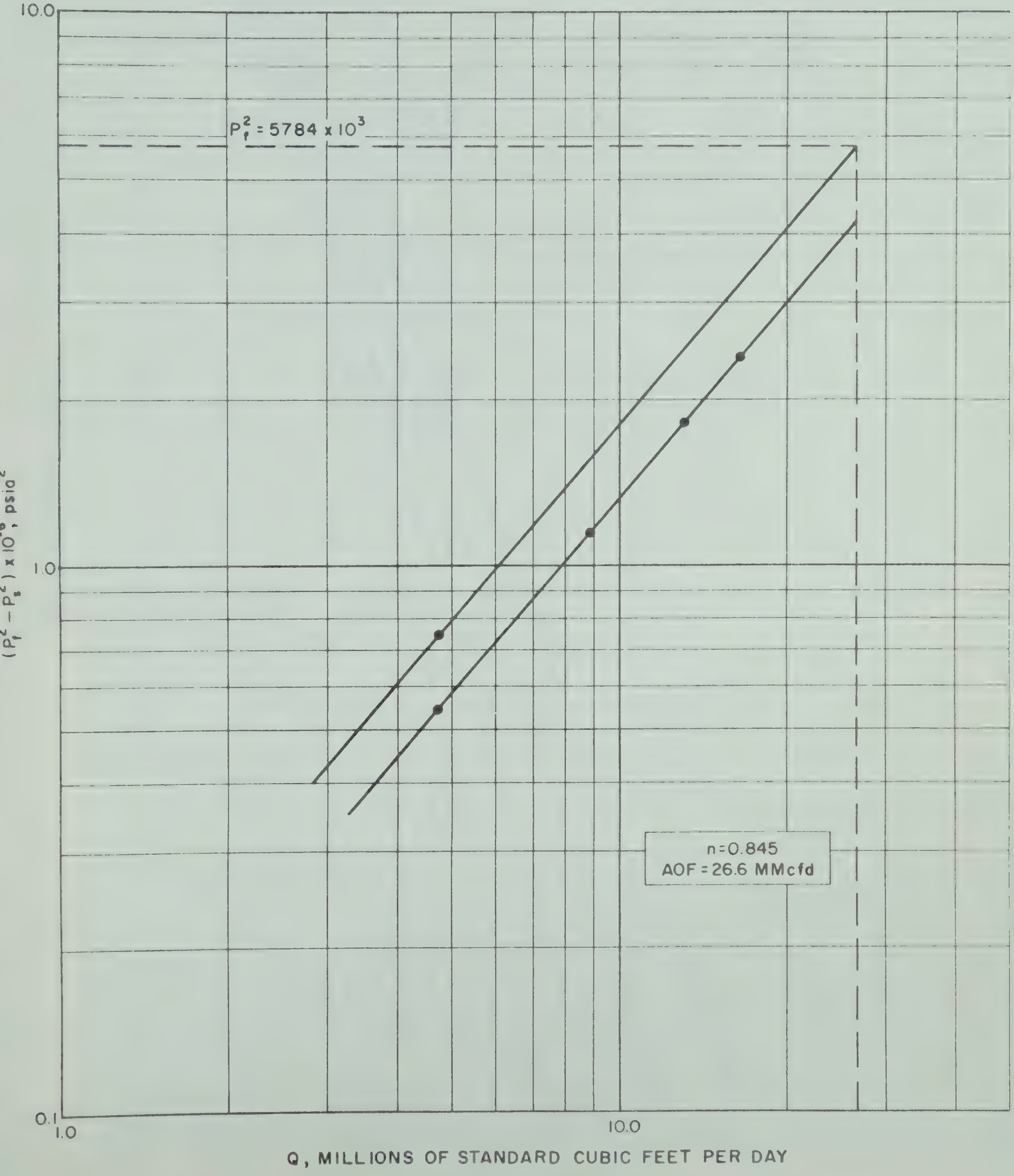


FIGURE 3-4 — MODIFIED ISOCHRONAL BACK PRESSURE TEST PLOT

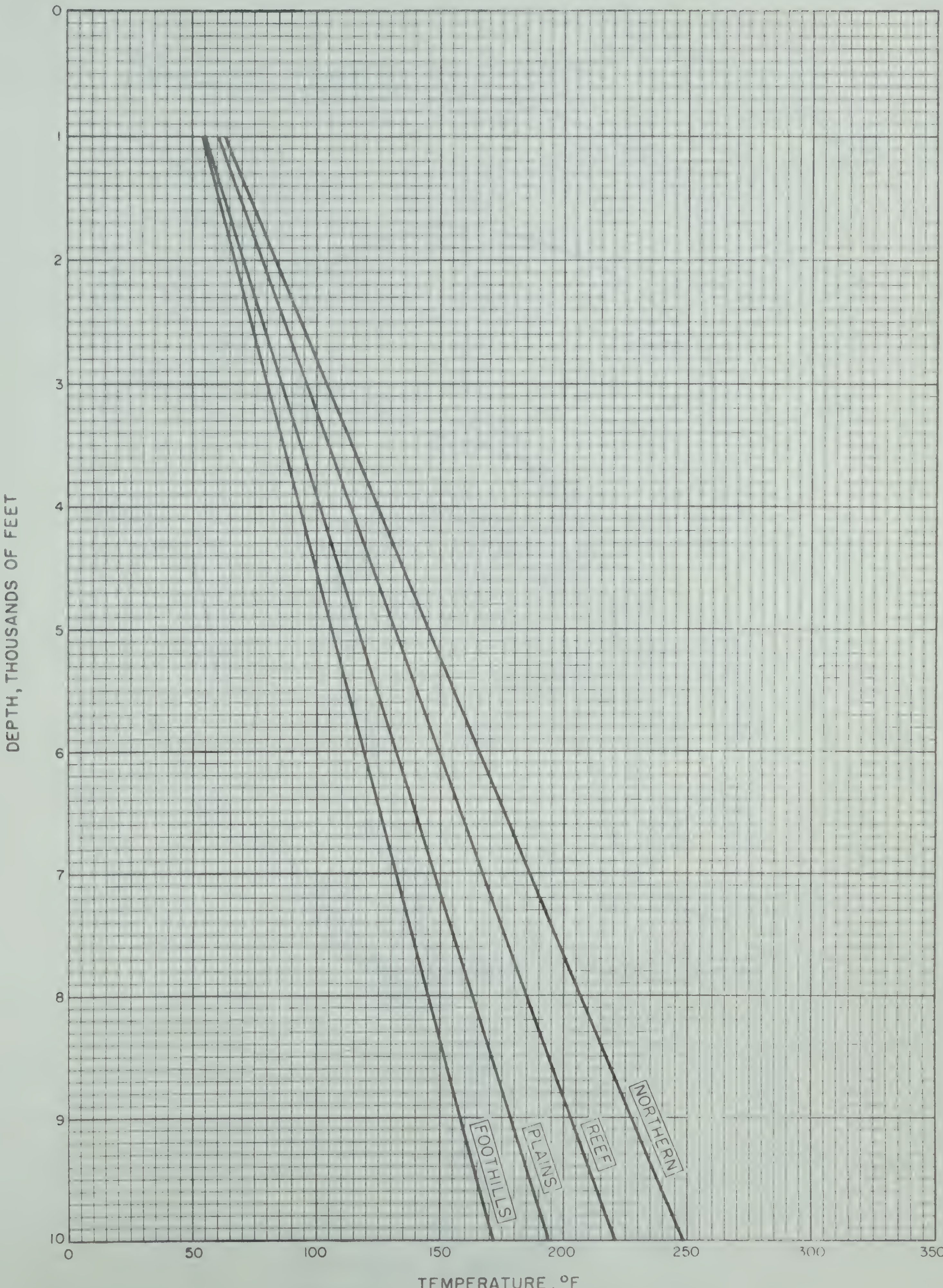


FIGURE 3-5 — TEMPERATURE GRADIENTS - PROVINCE OF ALBERTA.

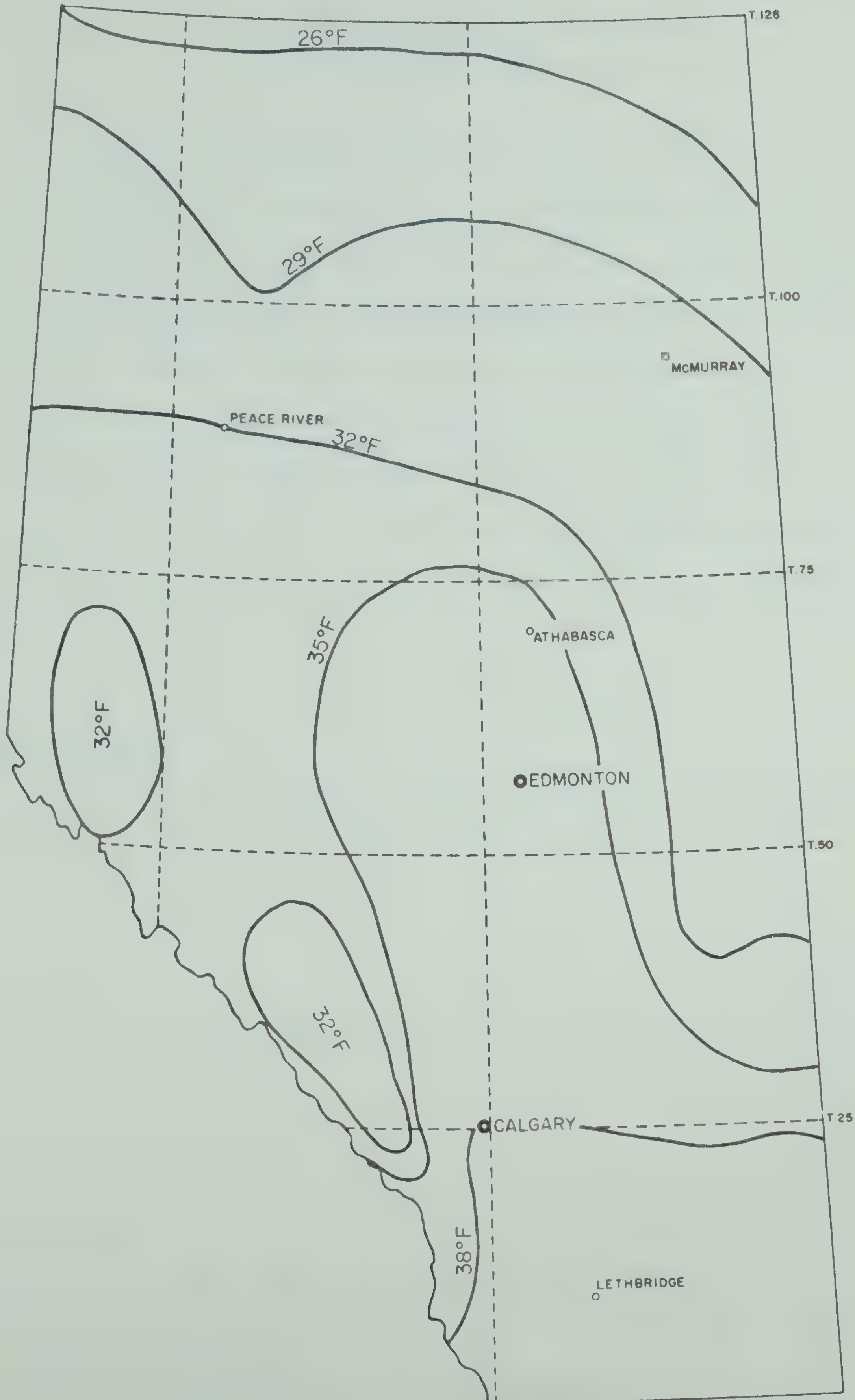


FIG. 3-6—MEAN ANNUAL SURFACE TEMPERATURES
PROVINCE OF ALBERTA.
(Redrawn from Beach, 5)

4. THE INTERPRETATION OF BUILD-UP AND DRAWDOWN DATA

Build-up and drawdown tests at gas wells are particularly useful in connection with productivity tests. Interpretation of these tests permits determination of the effective values of the combined reservoir and gas properties which are required for solution of the theoretical productivity equations. Moreover, the build-up tests may be used to determine the true static formation pressure from pressure-time observations taken over time periods shorter than would be required for the well to build up to the stabilized pressure. The stabilized pressure is necessary in the plotting and interpretation of any back pressure test.

Pressure Build-Up

The theory of pressure build-up is based on the unsteady-state flow theory for the multiple transient case. Equation (C-38), from Appendix C, may be written as follows:

$$\frac{P_f^2 - P^2_{(r,t)}}{P_f^2} = q_{D_1} P_{t_1} + (q_{D_2} - q_{D_1}) P_{t_2} + (q_{D_3} - q_{D_2}) P_{t_3} + \ldots +$$

$$(q_{D_n} - q_{D_{n-1}}) P_{t_n} \qquad \text{(C-38)}$$

where

t_1 = total elapsed time since the first flow rate began.

t_2 = total elapsed time since the second flow rate began.

t_n = total elapsed time since the nth flow rate began.

q_{D_1} = first dimensionless flow rate.

q_{D_n} = nth dimensionless flow rate.

P_{t_1} = dimensionless pressure change number evaluated at time t_1.

P_{t_n} = dimensionless pressure change number evaluated at time t_n.

The proper manner of conducting a pressure build-up test involves producing a well for a considerable period of time at an essentially constant flow rate. The well is then closed-in and pressure-time observations are recorded. If the constant producing rate corresponds to a dimensionless flow rate of q_{D_1}, then q_{D_2} and following q_D's to q_{D_n} are equal to zero and equation (C-38) when referring to the sandface reduces to

$$\frac{P_f^2 - P^2_{(r_s,t)}}{P_f^2} = q_{D_1}(P_{t_1} - P_{t_2}) \qquad (4\text{-}1)$$

As the pressure in a well builds up toward the shut-in pressure, there is a period during which the reservoir boundary effects are not felt and the reservoir acts as though infinite. Under these conditions, the dimentionless drawdown, P_t, may be expressed in the form of equation (C-36)

$$P_t = 1/2(\ln t_D + 0.80907) \qquad (C\text{-}36)$$

where t_D = dimensionless time.

Combining equations (C-36) and (4-1)

$$\frac{P_f^2 - P^2_{(r_s, t)}}{P_f^2} = \frac{q_{D_1}}{2} (\ln t_{D_1} - \ln t_{D_2}) \tag{4-2}$$

Since t_{D_1} corresponds to the total elapsed time since the first flow rate began (which is $t_f + \Delta t$), and t_{D_2} corresponds to the total elapsed time since the second flow rate began (Δt), equation (4-2) may be written as follows:

$$P_m^2 = P_f^2 - \frac{q_D P_f^2}{2} \ln \left\{ \frac{t_f + \Delta t}{\Delta t} \right\} \tag{4-3}$$

where

P_m = measured sandface pressure at any time $\left(P_{(r_s, t)}\right)$, psia.

P_f = shut-in formation pressure, psia.

q_D = dimensionless flow rate corresponding to the constant flow rate prior to shut-in.

t_f = period of flow prior to shut-in, hours.

Δt = shut-in time, hours.

From the form of equation (4-3), it follows that if the squares of the observed pressures, P_m^2, are plotted against the log of $(t_f + \Delta t)/\Delta t$, a straight line may be expected with the following slope:

$$\text{slope} = \frac{q_D P_f^2}{2} \tag{4-4}$$

If equation (C-33) for the dimensionless flow rate, is combined with equation (4-4), then the

$$\text{slope} = \frac{1.42 \times 10^6 u_a Z_a T_a Q}{2\ k\ h} \tag{4-5}$$

The permeability-pay product may be calculated from the slope of the straight line portion of the plot of P_m^2 versus log $(t_f + \Delta t)/\Delta t$, using the following equation:

$$kh = \frac{7.1 \times 10^5 \mu_a Z_a T_a Q}{\text{slope}} \qquad (4\text{-}6)$$

where

k = permeability, millidarcys.

h = pay thickness, feet.

μ_a = average viscosity, centipoise.

Z_a = average gas compressibility factor.

T_a = average temperature, ^oR.

Q = gas flow rate, millions of cubic feet per day at standard conditions of 14.65 psia and 60^oF.

and slope = $\dfrac{\text{change in } P_m^2 \text{ per cycle}}{2.303}$

One problem which arises in processing pressure build-up data, is the determination of the proper value of the time of flow, t_f, and the flow rate, Q, that should be used where a well has produced at various rates over a considerable period of time. In this situation, it is recommended that the stabilized flow rate be taken as the constant rate of flow just prior to shut-in. The "pseudo flow time" should then be determined by dividing this flow rate into the total production obtained over the time interval being investigated. (Where the insitu permeability is being calculated and the time of stabilization as calculated from equation (D-16), is less than the pseudo flow time, the time of stabilization should be used as the length of the flow period, t_f. This is not the case though, when build-up data is being used to calculate the static reservoir pressure.)

A further study of equation (4-3), would indicate that for an infinite reservoir, if the straight line were extrapolated to the end of an infinitely large shut-in time period, log $(t_f + \Delta t)/\Delta t = 1.0$, the static

formation pressure would be obtained. It is seldom that the pressure build-up of a well reflects infinite reservoir behavior during the final stages of build-up, and as a result this approach for determining the true static pressure is useful only in the case of a single well located in the approximate centre of a large pool, from which little production has been taken. Further development of this theory towards the determination of the static reservoir pressure in presence of boundary effects is complex Many authors have presented methods applicable under particular circumstances, some of which will be discussed later in this section.

Pressure Drawdown

A pressure drawdown test consists of measuring and recording the declining sandface pressure as a function of time, essentially from stabilized shut-in conditions, as the well is produced at a constant flow rate. The method is particularly useful early in the producing life of the first few wells completed in a reservoir, however, it may also be applied to wells which have been shut-in for a considerable period of time.

If equation (C-34)

$$P_t = \frac{1}{q_D} \frac{P_f^2 - P^2_{(r,t)}}{P_f^2} \tag{C-34}$$

is combined with equation (C-36), the following equation results

$$P_m^2 = P_f^2 - 0.4045\, q_D\, P_f^2 - \frac{q_D\, P_f^2}{2} \ln t_D \tag{4-7}$$

Equation (4-7) may be written as

$$P_m^2 = \text{constant} - \frac{q_D\, P_f^2}{2} \ln t_D \tag{4-8}$$

If equation (4-8) is combined with equation (C-32) the relationship of t_D to t

$$t_D = \frac{2.634 \times 10^{-4} k\, P_a t}{\mu_a \phi\, r_s^2} \qquad \text{(C-32)}$$

where

k	=	permeability, md.
P_a	=	average pressure, psia.
t	=	time, hours.
μ_a	=	average viscosity, cp.
ϕ	=	gas filled porosity, fraction.
r_s	=	effective well bore radius, feet.

the following equation results

$$P_m^2 = \text{constant} - \frac{q_D\, P_f^2}{2} \ln t \qquad \text{(4-9)}$$

The form of equation (4-9) leads to the conclusion that a plot of the observed sandface pressures squared, P_m^2, versus the log of the time of the observation, t, will result in a straight line relationship with a slope equal to $q_D\, P_f^2/2$. Combining the value of the slope and equation (C-33), the following relationship results.

$$kh = \frac{7.1 \times 10^5\, \mu_a Z_a T_a Q}{\text{slope}} \qquad \text{(4-10)}$$

In this case, the flow rate, Q, should be taken as the average flow rate during the drawdown test.

Determination of Static Reservoir Pressure from Build-Up Tests

Many methods are available for using pressure build-up data

to estimate the static formation pressure. Some of the better known and earlier papers on this subject are those of Muskat (62), Horner (40), Arps and Smith (2) and Miller, Dyes and Hutchinson (61). Many other papers have been presented which suggest modifications to the various methods of interpreting pressure-time data or which investigate special problems encountered in conducting build-up tests (26)(44)(52)(58)(59)(65)(71)(89). In addition, many authors have discussed the determination of well bore effects (commonly referred to as skin effect) from build-up and flow data (3)(18)(43)(76)(80). These effects are very important in understanding back pressure tests.

It is not considered appropriate to deal here in full detail with the many methods for the conduct and interpretation of build-up tests. For illustrative purposes, two of the basic methods are presented. They are those proposed by Muskat and by Horner (as modified by Tracy). Applications of the various methods to many test data have led to the conclusion that each method has advantages under certain circumstances. These circumstances may be related to boundary conditions, formation geometry and homogeneity, rock characteristics, and to general conditions prior to and during the conduct of the test. Given adequate data, it is believed, that either the Muskat or Horner method should give reasonable results with respect to interpreting any pressure-time data taken under normal circumstances.

The Muskat Method

The Muskat method has been developed on the basis of the theory of the unsteady-state flow of a fluid from a bounded reservoir towards a well, starting from the equivalent of equation (C-34). It involves a trial and error solution in which the static pressure is guessed, and the log of the assumed static pressure minus the measured pressure is plotted for

each pressure-time observation. If the choice of static pressure is correct, the resulting plot will be a straight line. If the estimate of the static pressure is too high the plotted points will curve upwards and if it is too low the plotted points will curve downwards. (The method was developed for use with oil wells and when applied to gas wells should be in the form of a semi-log plot of the difference of the pressures squared versus time.)

Larson (52) has shown that although the true equation for the build-up curve of a bounded well is not the same as the equation of the Muskat curve for early times, it does become linear and identical to the Muskat curve at dimensionless times greater than 0.08. Accordingly, the method should only be employed with data representing dimensionless times greater than 0.08. Larson also modified the method somewhat to permit the determination of the permeability-pay product of the formation.

The main advantage of the Muskat method (with respect to determination of static pressures), is the relatively good accuracy which can be obtained with a simple calculation. Other advantages are that it does not rely on an extrapolation of the pressure-time curve, and that the time interval for which the method is valid may be easily calculated. The fact that the method can be applied only for dimensionless times greater than 0.08, results in an answer which is not affected by after-production which has been shown to exist only for dimensionless times less than approximately 0.02 (61). (After-production is the continued production into the well bore due to the compressibility of the fluids in the well bore and results from closing a well in at the surface rather than at the sand-face.)

A major disadvantage associated with the Muskat method is that the pressure-time data must be obtained over a longer shut-in period than with several other methods. Also, the method does not appear to be practical for use in very tight reservoirs. In these situations the very slow

nature of the build-up makes it difficult to judge whether the plot corresponding to a particular assumed pressure is truly a straight line.

The recommended procedure for conducting a build-up test so that the data can be used for determining the static pressure by the Muskat method is as follows.

1. Prior to shut-in, produce the well for a reasonable period of time at approximate stabilized conditions.

2. Calculate the time corresponding to a dimensionless time of 0.08, from equation (C-32). The permeability used in this calculation should be estimated from core analysis and/or knowledge of the formation. Upon completion of the build-up the accuracy of the estimate may be checked against the actual effective permeability.

3. Record pressure observations during the initial stages of the build-up. This will enable one to see the upwards curvature which always occurs for dimensionless times less than 0.08, and will help with the final interpretation of the plot.

At dimensionless times greater than 0.08, regular pressure observations must be recorded. Six to ten data points should be taken over a period equivalent to a total dimensionless time of about 0.16.

4. Estimate an appropriate static pressure, P_f. The first guess may be based on previous knowledge of the formation pressure and on the behavior of the recorded build-up. Calculate values of the estimated static pressure squared minus the measured pressure squared, $P_f^2 - P_m^2$, for each time of observation, and plot these values on the log axis versus the time on the linear axis (see Figure 4-5). If the portion of the line beyond $t_D = 0.08$ curves downwards a higher estimate of the static pressure is made. If it curves upwards a lower estimate is made. This trial and error procedure is repeated until the plotted relationship becomes a

straight line. The final assumption will reflect the correct static formation pressure.

The Horner Method

Horner has developed a method for determining the static pressure of an oil well by plotting the observed pressure against the log of $(t_f + \Delta t)/\Delta t$. When this method is modified to apply to gas wells, the plot is identical to that discussed earlier in connection with determining the permeability and pay from a build-up. (The modification of the method for a gas reservoir is based on work by Aronofsky and Jenkins (1) who compared the behavior of the pressure transient for a gas well to that of an oil well.)

The basic theory of the Horner build-up method is similar to that developed earlier in this section, except that a correction for boundary effects expected in a finite reservoir has been included. It has been mentioned that the infinite reservoir case may be applied to a new well in a finite reservoir. This method now deals with wells in reservoirs from which production has been taken. In developing the method, it is assumed that a previous pressure survey has been conducted on the well under consideration. In this case, where boundary effects are felt, the equation of flow (as developed by Horner and modified for gas wells) may be expressed as

$$P_m^{\ 2} = P_p^{\ 2} - \frac{q_D P_f^{\ 2}}{2} \left(\ln \frac{t_f + \Delta t}{\Delta t} + y\ (u)\right) \qquad (4\text{-}11)$$

P_m = the sandface pressure measured during a build-up test, psia.

P_p = the pressure at the time of the previous survey, psia.

u = a variable which depends on reservoir and flow characteristics (reservoir permeability, porosity, pressure; gas temperature and viscosity; and on the radius to the external boundary).

$y(u)$ = a function of u defined as follows

$$y(u) = E_i\ (-u) + \frac{1}{u} e^{-u}$$

Horner presented a plot of y(u) for a large range of values of u. Tracy (89) has modified this plot as presented in Figure 4-1. The modification by Tracy is actually a plot of u versus y(u)/2.303.

It follows that if the square of the observed pressures are plotted versus log $(t_f + \Delta t)/\Delta t$, equation (4-11) may be expressed as follows

$$P^{*2} = P_p^{\ 2} - \frac{[(\text{slope of the plot})y(u)]}{2.303} \qquad (4\text{-}12)$$

where P^* = the extrapolation of the straight line portion of the plot to log $(t_f = \Delta t)/\Delta t$ equal 1.0. In the case of a finite reservoir this is a false value of the final static pressure (thus designated as P*).

From a knowledge of P*, P_p, and the slope of the plot, y(u)/2.303 can be determined. Values of u corresponding to y(u)/2.303 are shown in Figure 4-1

For the case of an infinite shut-in time and where the square of the observed pressures are plotted versus log $(t_f + \Delta t)/\Delta t$, equation (4-11) may be written as

$$P_f^2 = P_p^2 - \frac{\text{slope of the plot}}{2.303\ u} \qquad (4\text{-}13)$$

It follows that the correct static reservoir pressure, P_f, may be calculated from a knowledge of the pressure at the time of the previous survey, the slope of the plot of the observed pressures squared versus log $(t_f + \Delta t)/\Delta t$, and the previously determined value of u.

The recommended procedure for conducting a build-up test and calculating the static formation pressure by this method is as follows.

1. Flow the well at a constant rate for some time prior to shut-in.

2. Calculate the producing time, t_f, by dividing the total production over the time interval considered, by the constant flow rate prior to shut-in. The interval being considered is the time elapsed since the previous pressure survey.

3. Record pressure-time observations during the shut-in period.

4. Plot the squares of the observed pressures against the log $(t_f + \Delta t)/\Delta t$.

5. Extrapolate the straight line portion of the plot (from which the permeability-pay product may be determined) to log $(t_f + \Delta t)/\Delta t = 1.0$ to determine P^{*2}.

6. Calculate the slope of the straight line portion of the plot.

7. Calculate $\frac{y\ (u)}{2.303}$ from equation (4-12).

8. Determine u from Figure 4-1.

9. Calculate P_f from equation (4-13).

The Muskat and the Horner methods for determining the static formation pressure; and the determination of the permeability-pay product from build-up or drawdown data are illustrated in the following examples.

Example 4-1

A new well in a newly discovered field is produced for 48 hours and then closed in. The cumulative gas production is 17,110,000 cubic feet, and the rate of flow immediately prior to shut-in is 8.5 million cubic feet per day. Drawdown data are recorded during the flow period, and build-up data recorded for the first 24 hours after shut-in. (These represent actual data taken on a gas well in the Province of Alberta.)

If the following reservoir and flow characteristics are known, calculate the permeability-pay product from both the drawdown and build-up data.

$\mu_a = 0.02$ cp

$Z_a = 0.88$

$T_a = 560^{\circ}R$

Drawdown Data

Time of Flow t - hours	Measured Pressure P_m - psia	$(P_m)^2 \times 10^{-5}$ $(psia^2)$
0.1	983	9.66
0.2	958	9.18
0.3	944	8.91
0.6	918	8.43
1.0	899	8.08
1.5	885	7.83
3.0	858	7.36
6.0	833	6.94
10.0	814	6.63
16.0	798	6.37
26.0	780	6.08
36.0	768	5.90
48.0	758	5.75

A plot of P_m^2 versus log t is then prepared (see Figure 4-2). From equation (4-10)

$$kh = \frac{7.1 \times 10^5\ (0.02)(0.88)(560)(8.5)}{(144{,}000)/2.303}$$

$$kh = 951 \text{ md-ft.}$$

Build-up Data

Time of Shut-In Δt - hours	Measured Pressure P_m - psia	$(P_m)^2 \times 10^{-6}$ (psia^2)	$\frac{t_f + \Delta t}{\Delta t}$
2	1002	1.004	25
3	1009	1.018	17
4	1017	1.034	13
5	1026	1.053	10.6
6	1033	1.067	9.0
8	1042	1.086	7.0
11	1052	1.107	5.4
13	1061	1.126	4.7
16	1066	1.136	4.0
20	1072	1.149	3.4
24	1079	1.164	3.0

$$t_f = 48 \text{ hours}$$

A plot of P_m^2 versus log $(t_f + \Delta t)/\Delta t$ is prepared (see Figure 4-3).

From equation (4-6)

$$kh = \frac{7.1 \times 10^5 \,(0.02)(0.88)(560)(8.5)}{(195{,}000)/2.303}$$

$$kh = 702 \text{ md-ft.}$$

The agreement between the permeability-pay product as calculated from a drawdown and a build-up test is considered reasonable.

Example 4-2

A well which produced 1,695 million cubic feet of gas since its last pressure survey is shut-in. The static pressure at the time of the previous survey was 2250 psia. The well was produced at a rate of 7.5 million cubic feet per day immediately prior to being closed in. The well is located three-quarters of a mile from each of two neighboring wells. The following reservoir and gas characteristics are known.

$$T_a = 620^\circ R$$

$$\mu_a = 0.018 \text{ cp}$$

Z_a = 0.81

ϕ = 0.05

h = 60 feet

Determine the static formation pressure given the following build-up data. (These represent actual data taken on a gas well in the Province of Alberta.)

Build-up Data

Time of Shut-In Δt - hours	Measured Pressure P_m - psia	$(P_m)^2 \times 10^{-3}$ (psia^2)	$\dfrac{t_f + \Delta t}{\Delta t}$
2	2031	4125	2710
3	2041	4166	1810
5	2054	4219	1086
7	2061	4248	776
10	2071	4289	543
15	2082	4335	363
20	2089	4364	272
24	2092	4376	227
30	2098	4402	182
36	2102	4418	152
42	2107	4439	130
48	2110	4452	114
60	2114	4469	91
72	2118	4486	76
84	2121	4499	66
96	2125	4516	58
108	2128	4528	51
120	2131	4541	46
132	2133	4550	42

(a) Horner Method

$$t_f = \frac{1695}{7.5}(24) = 5424 \text{ hours}$$

From equation (4-6)

$$k = \frac{7.1 \times 10^5\ (0.018)(0.81)(620)(7.5)}{(60)(245,000)/2.303} = 7.5 \text{ md.}$$

The plot (Figure 4-4) is extrapolated to log $(t_f + \Delta t)/\Delta t$

equals 1.0, where

$$P^{*2} = 4,950,000$$

From equation (4-12)

$$4,950,000 = (2250)^2 - \frac{245,000\ y(u)}{2.303}$$

$$\frac{y(u)}{2.303} = 0.46$$

From Figure 4-1

$$u = 0.38$$

Substituting in equation (4-13)

$$P_f^{\,2} = (2250)^2 - \frac{245,000}{(0.38)(2.303)}$$

$$P_f = 2,187 \text{ psia}$$

The static pressure calculated by the Horner method is 2,187 psia.

(b) Muskat Method

The time corresponding to a dimensionless time of 0.08 is calculated from equation (C-32). (In this instance, equation (C-32) has been adjusted to reflect the external boundary radius rather than the well bore radius.)

$$t = \frac{(0.018)(0.05)(1980)^2(0.08)}{2.634 \times 10^{-4}(7.5)(2200)} = 65 \text{ hours}$$

So the Muskat plot should result in a straight line after approximately 65 hours. Figure 4-5 is such a plot. The assumed static pressures are 2,140 psia, 2,160 psia and 2,180 psia respectively. The plot illustrates how difficult it is to determine the precise pressure using this method, however, it does indicate that the pressure is in the order of 2,180 psia, as was obtained by the Horner method.

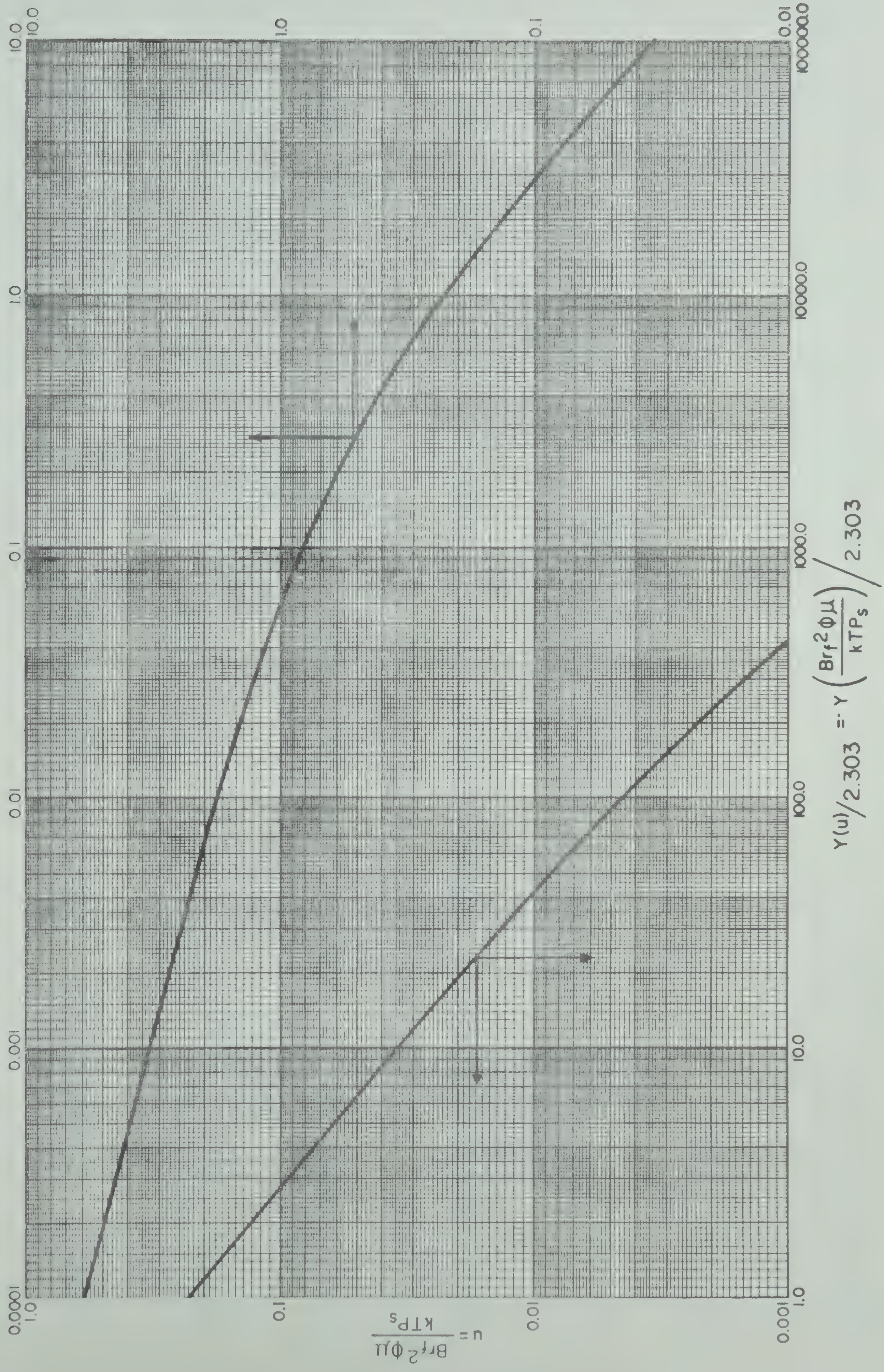


FIGURE 4-1 — Y FUNCTION CORRELATION (Redrawn from Tracy, 89).

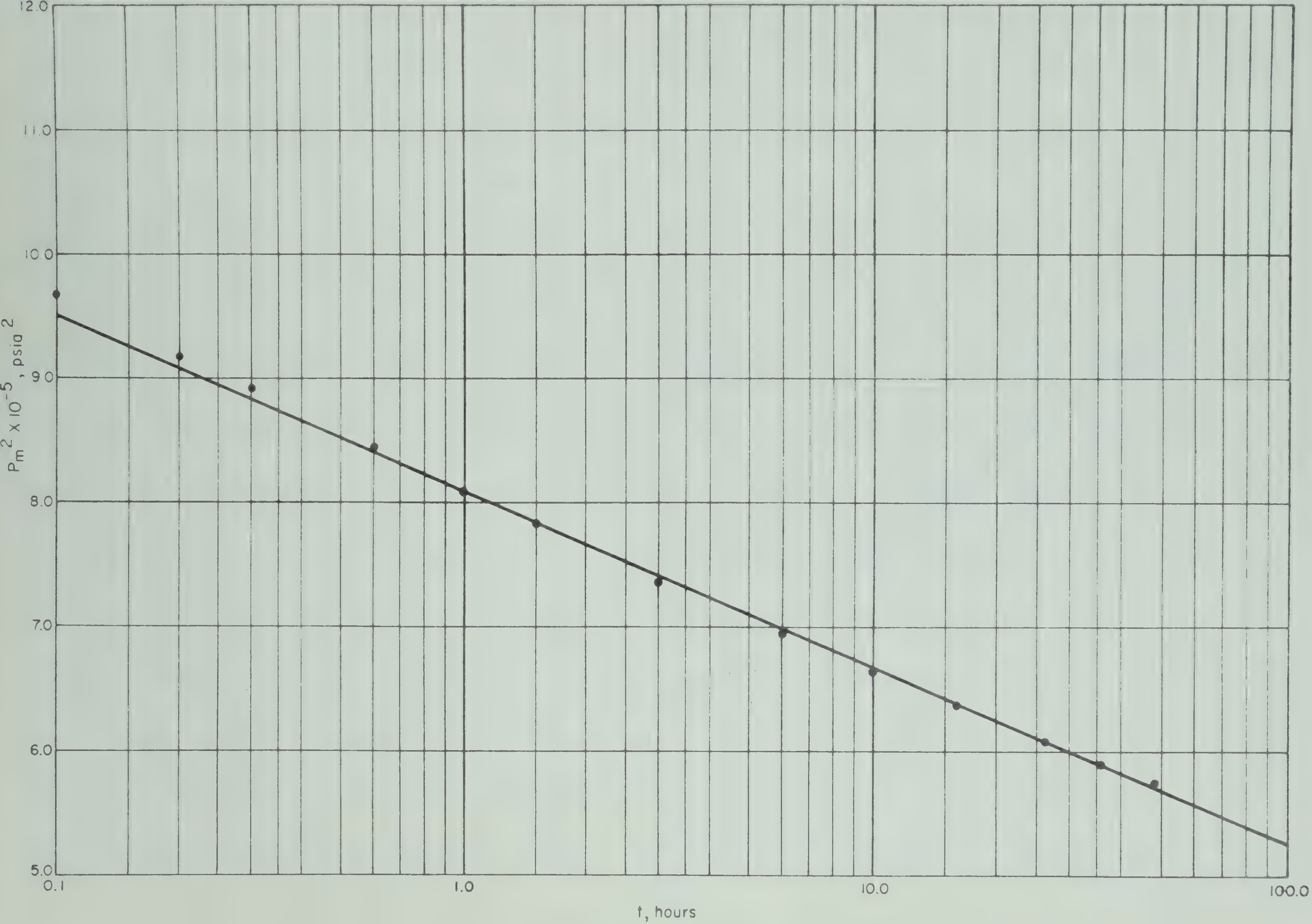


FIGURE 4-2 — PRESSURE DRAWDOWN DATA.

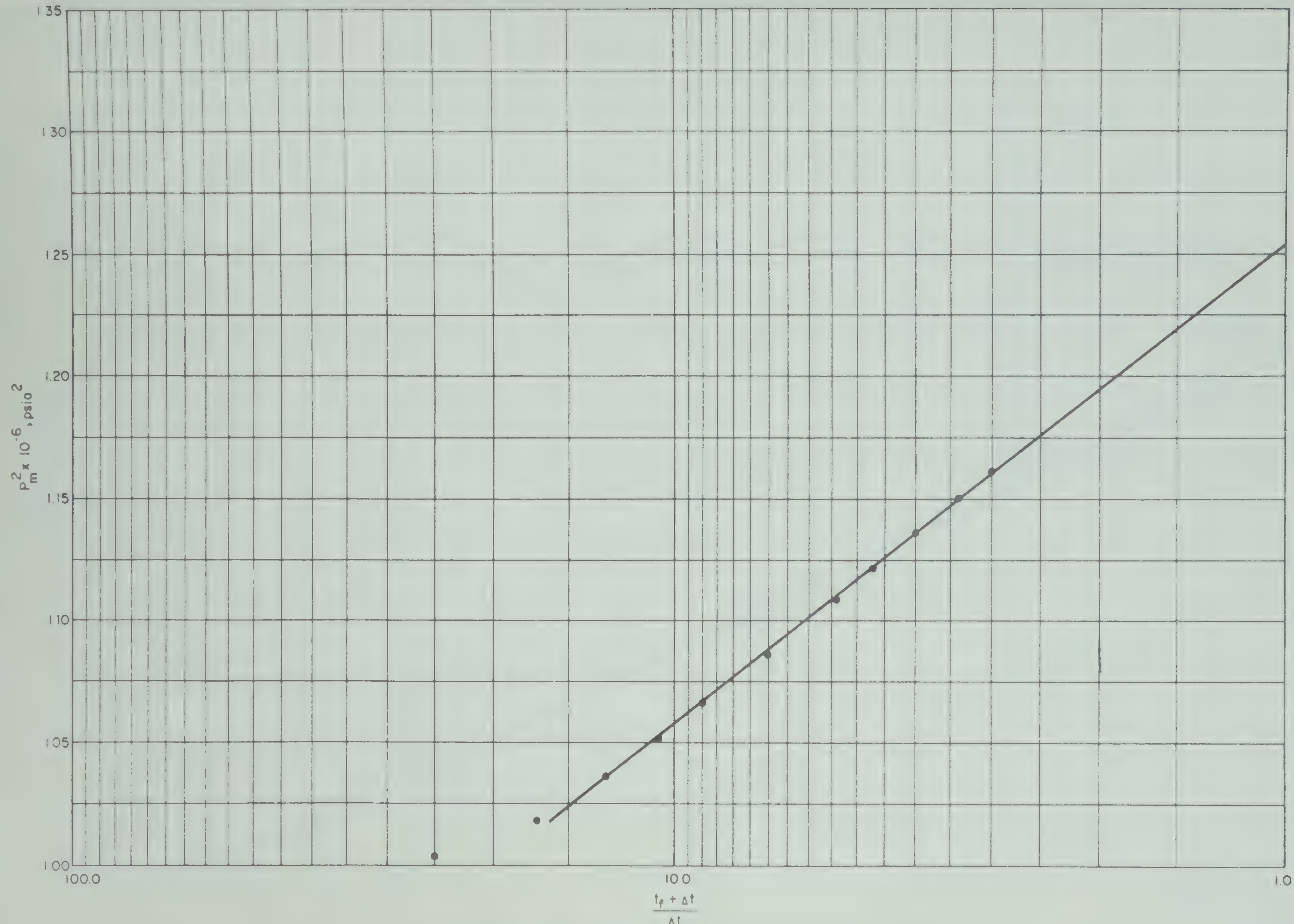


FIGURE 4-3 — PRESSURE BUILD-UP DATA.

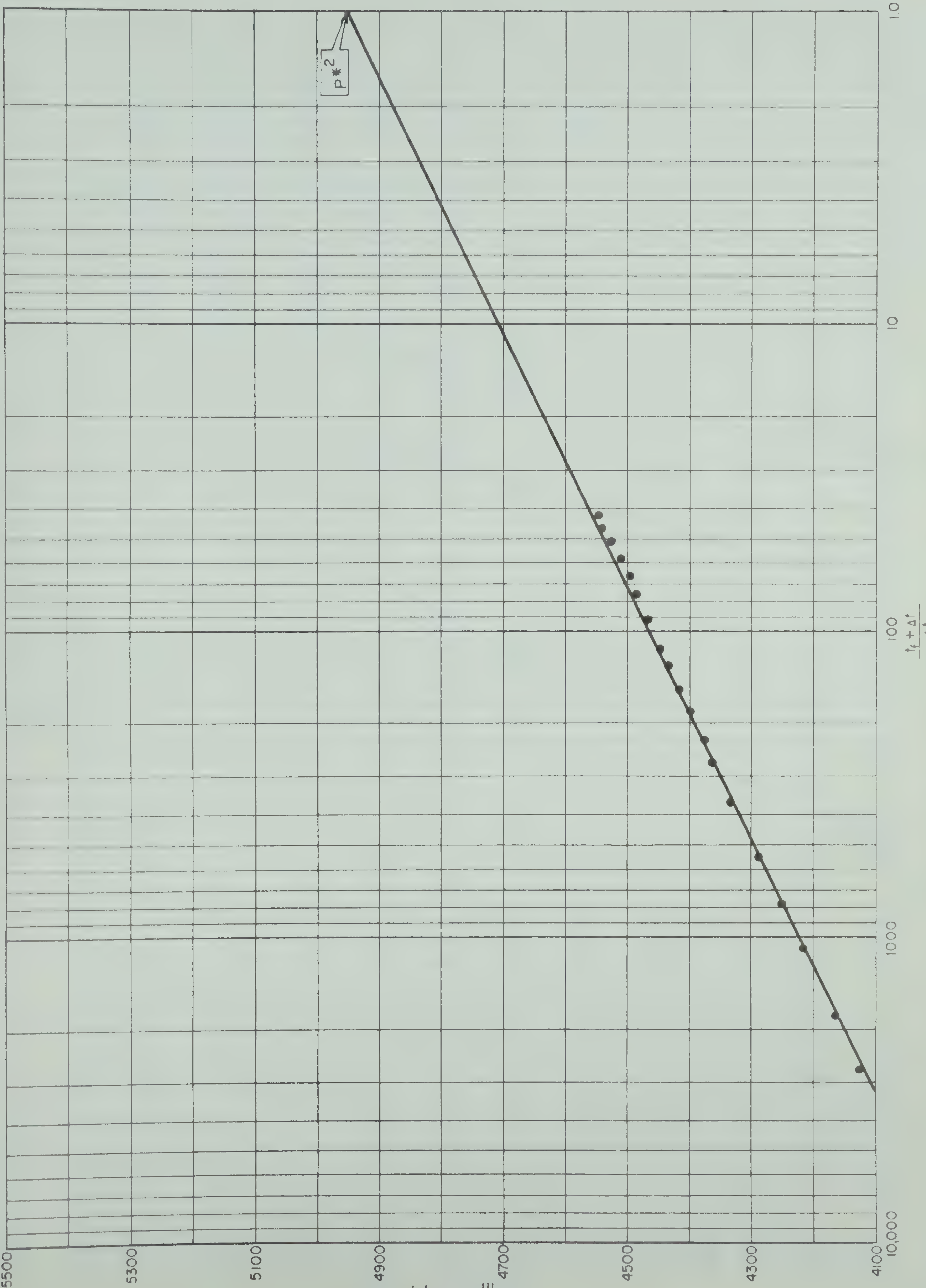


FIGURE 4-4 — HORNER PRESSURE BUILD-UP PLOT.

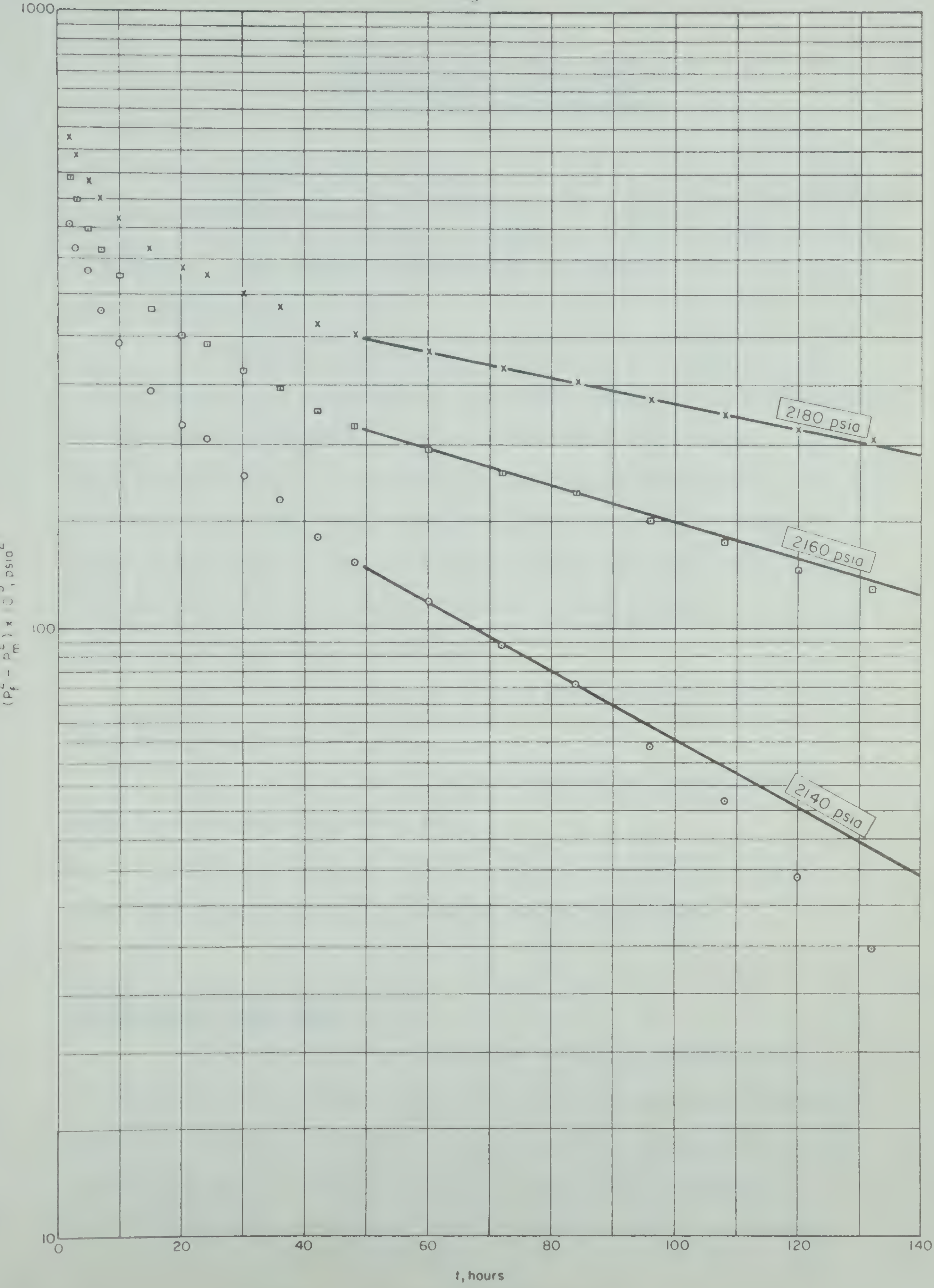


FIGURE 4-5 — MUSKAT PRESSURE BUILD-UP PLOT.

5. THE ESTIMATION OF FLOW BEHAVIOR FROM THEORY AND LIMITED DATA

The previous sections dealt in some detail with the determination of the back pressure relationship for a gas well through a series of actual flow tests. This section presents methods of estimating the flow capacity from a limited amount of data.

The problem of calculating the anticipated behavior of a gas well on the basis of reservoir and flow characteristics has been discussed by many authors (14)(19)(31)(35)(39)(46)(70)(80)(84)(85)(86)(88). Only the simplest theoretical approach to calculating the back pressure relationship will be discussed here, although mention will be made of some of the shortcomings of the theory and of several of the empirical methods which have been developed in an attempt to solve related problems. Included will be theoretical methods for estimating:

(i) the stabilized position of a back pressure curve from an isochronal multi-point flow test,

(ii) the slope and stabilized position of a back pressure curve from a single flow point, and

(iii) the slope and position of a stabilized back pressure curve calculated entirely from reservoir and gas properties.

Positioning a Back Pressure Curve to Stabilized Conditions

It has been demonstrated how the final flow rate of an isochronal test may be extended to position the back pressure line to reflect stabilized conditions. This positioning may also be accomplished through calculations based on the theory of gas flow presented in Appendix C. These calculations, however, are based on the knowledge of the interwell

characteristics of the formation and ideally should be employed only when such information has been either obtained through a properly conducted build-up or drawdown test or can be accurately estimated. In addition, a knowledge of the true static formation pressure and of the time required to attain adequate stabilization is necessary. The detail of estimating this time is included in Appendix D, but in general may be taken as the lesser of 15 days or the theoretical time to stabilization as calculated by equation (D-16). (It is recognized that the true static formation pressure can be calculated from data taken over a time period of less than 15 days. In fact, a maximum of only 6 days of build-up data is normally sufficient to calculate the reservoir pressure.)

The following is a step by step trial and error procedure which results in the positioning of a back pressure line of known slope to reflect stabilized flow conditions. The method assumes that data of an isochronal test is available, however, it is based on only one of the series of flow tests. The particular flow rate is selected for its reliability, and should be within the limits of laminar flow.

1. Calculate from equation (C-32) the dimensionless time for the sandface, corresponding to the short term flow test.

$$t_D = \frac{2.634 \times 10^{-4}\, k\, P_a t}{\mu_a\, \phi\, r_s^2} \tag{C-32}$$

where

k = permeability, md.

P_a = average pressure, psia.

t = time, hours.

μ_a = average viscosity, cp.

ϕ = gas filled porosity, fraction.

r_s = effective well bore radius, ft.

To accomplish this, the average pressure during the short term flow period is determined and the porosity, average viscosity of the flowing gas and the effective well bore radius are estimated on the basis of the best information available. The well radius may simply represent the size of the casing, or may incorporate adjustment for well bore damage based on either build-up, drawdown, or drill stem test data (76)(85)(90)(95). The effective permeability should be based on a build-up or drawdown test, or lacking one of these may be estimated from core analysis.

2. Determine the dimensionless drawdown, P_t, in accordance with the appropriate boundary conditions, from Figure C-2.

3. Calculate the dimensionless flow rate, q_D, for the short term flow period, from equation (C-34).

$$P_t = \frac{1}{q_D}\left(\frac{P_f^2 - P_s^2}{P_f^2}\right) \qquad \text{(C-34)}$$

where

P_t = dimensionless drawdown.

q_D = dimensionless flow rate.

P_f = formation pressure, psia.

P_s = flowing sandface pressure, psia.

In the definition of the dimensionless flow rate by equation (C-33)

$$q_D = \frac{1.42 \times 10^6 \mu_a Z_a T_a Q}{kh\, P_f^2} \qquad \text{(C-33)}$$

where

μ_a = average viscosity, cp.

Z_a = average compressibility factor.

T_a = average temperature, ^oR.

Q = gas flow rate, millions of cubic feet per day at 14.65 psia and 60^oF.

k = permeability, md.

h = pay, ft.

P_f = formation pressure, psia.

it is obvious that only the viscosity and compressibility factor will change with an increase in the duration of the flow test. Moreover, the variation in these gas properties will be small if the change in the pressure is not too great, so the value of, q_D, may be considered constant.

4. Calculate the theoretical time to attain an acceptable degree of stabilization corresponding to a radius of drainage of one-half mile, from equation (D-16).

$$t_s = 1000 \frac{r_f^2 \mu_a \phi}{kP_f} \qquad \text{(D-16)}$$

where

t_s = time to stabilization, hours.

r_f = exterior boundary radius, ft.

μ_a = average viscosity, cp.

ϕ = gas filled porosity, fraction.

k = permeability, md.

P_f = formation pressure, psia.

5. Assume a stabilized pressure, P_s, calculate P_a, and from equation (C-32), calculate the dimensionless time corresponding to the lesser of the time clculated on item 4 or 15 days.

6. From Figure C-2, determine a P_t corresponding to the time of stabilization.

7. From equation (C-34) and the knowledge that the dimensionless flow rate is constant, determine the sandface pressure. Check this against the value assumed in step 5, and if the difference is significant, repeat steps 5 to 7.

The stabilized value of $P_f^2 - P_s^2$ is then plotted against the constant flow rate, and a line having the slope of the isochronal line drawn through the point. The stabilized coefficient 'C' may readily be determined from the line.

The major drawback to this approach, is the difficulty experienced in accurately estimating certain reservoir properties, such as the effective well bore radius and the effective permeability. Another problem is related to turbulence. If the slope of the isochronal line is other than unity and thus reflects turbulence, it is logical to expect that an extension of the flow period to the time of stabilization would reduce the turbulence and therefore the slope of the line. This problem may be accounted for theoretically, but, is not serious under normal conditions and the method outlined should give reasonable results, particularly if applied to a short test where the flow in the reservoir is mainly laminar.

Carter, Miller and Riley (14), Cornell (19), Haymaker, Binckley and Burgess (39), Poettmann and Schilson (70), Swift and Kiel (84), Smith (80), Tek, Grove and Poettmann (85), and Winestock and Colpitts (96), have published methods for determining the stabilized coefficient on the basis of a short time flow test. Each of these methods is in general, based on the flow theory previously presented. Most methods also incorporate empirical relationships which have been developed to overcome the complexity associated with a purely theoretical approach.

Establishing a Back Pressure Curve
From a Single Flow Point

Theoretically, the slope of a stabilized back pressure line may be calculated from the combined steady-state laminar-turbulent flow equation. This equation relates the total pressure drop associated with flow, to the flow rate and to certain properties of the reservoir and the flowing gas. According to this theory, which assumes complete homogeneity throughout the reservoir, the slope of a stabilized back pressure curve should be unity under laminar flow conditions and any departure from this slope should be ascribed to turbulence. (Turbulence as here used includes all deviations from strict laminar flow and specifically those due to eddy currents, kinetic energy degradation, as discussed by Houpeurt (41), expansion and contraction losses, and the like. The turbulence is concentrated in a zone near the well bore where flow velocities are greatest.) Experience, however, indicates that most gas wells, even when tested under completely stabilized conditions, yield a back pressure line of steeper slope than may be explained by using the laminar-turbulent flow equation which includes the effects of turbulence. It seems clear that this additional deviation from steady-state theory may only be explained through serious deviations from the concept of complete uniformity of reservoir properties caused by variations in permeability, presence of channels and fractures, partial impregnation of the formation with drilling fluids, and variable saturation with respect to liquids.

Notwithstanding the above mentioned problems, it may at times be necessary to estimate, by calculation, the slope of the back pressure relationship. In this situation, if it is assumed that the formation is reasonably uniform throughout, and that the effective well bore is approximately circular, equation (C-15) may be used to calculate the slope of the back pressure curve.

$$P_f^{\,2} - P_s^{\,2} = \frac{1.418 \times 10^6}{kh} \mu_a T_a Z_a Q \ln\left(\frac{r_f}{r_s}\right) + \qquad \text{(C-15)}$$

$$\frac{3.140 \times 10^{-6}}{h^2} \beta\ G\ Z_a T_a Q^2 \left(\frac{1}{r_s} - \frac{1}{r_f}\right)$$

where

P_f = formation pressure, psia.

P_s = flowing sandface pressure, psia.

k = permeability, md.

h = pay, ft.

μ_a = average viscosity, cp.

T_a = average temperature, ^oR.

Z_a = average compressibility factor.

r_f = exterior boundary radius, ft.

r_s = effective well bore radius, ft.

β = turbulence factor.

Q = gas flow rate, millions of cubic feet per day at 14.65 psi and 60^oF.

Govier (35) has presented a convenient means of using this equation through the introduction of the turbulence contribution factor. As discussed in Appendix C, the laminar-turbulent radial flow equation may be expressed in the following form:

$$P_f^{\,2} - P_s^{\,2} = a\ Q\ F_t \qquad \text{(C-22)}$$

where F_t, the turbulence contribution factor, is described by the following equation.

$$F_t = 1 + \frac{0.09101\ G\ Q}{\mu_a k^{1/3} r_s\ h} \ \frac{1}{\ln\ (r_f/r_s)} \qquad \text{(C-23)}$$

It should be noted at this time, that the above equations have been developed on the basis of steady-state theory and that flow in a reservoir is an unsteady-state phenomenon. Nonetheless, use of steady-state equations is satisfactory provided it is assumed that flow originates at the apparent or steady-state radius of drainage. Janicek and Katz (45) have suggested a definite ratio of the steady-state radius of drainage and the effective radius of drainage, (0.749 when the effective radius is moving and 0.606 when it is stationary).

The following is the recommended procedure for calculating a stabilized back pressure curve from a single flow test.

1. Plot the difference of the static and the flowing pressures squared, $(P_f^2 - P_s^2)$, versus the flow rate, Q, on log coordinates.

2. Position the point to reflect stabilized flow conditions in the reservoir by the method presented earlier in this section, or by some other acceptable approach.

3. Calculate the turbulence contribution factor corresponding to the flow rate, Q, from equation (C-23), and correct the stabilized point as determined under item 2, to reflect laminar flow conditions. The permeability used in this calculation should be the effective permeability as determined from a build-up or drawdown test, and the effective well bore radius may be adjusted to incorporate any damage or stimulation that may have occurred during drilling or subsequent operations at the well.

4. Draw a line representing laminar flow conditions (of slope equal to unity) through the stabilized laminar flow point.

5. Calculate the turbulence contribution factor as a function of Q, from equation (C-23).

6. Select a series of flow rates representing the range over which the well will normally produce and determine the value of $(P_f^2 - P_s^2)$

for laminar flow conditions from the line of slope equal to unity for each of the selected flow rates.

7. Calculate the turbulence contribution factor for each of the flow rates.

8. Correct the previously determined values of $(P_f^2 - P_s^2)_{laminar}$ for turbulence by multiplication with the appropriate turbulence contribution factors calculated in step 7.

9. Plot the adjusted values of $(P_f^2 - P_s^2)$ on log coordinates against the selected values of Q. The best fit straight line is drawn through these points. This line should represent a stabilized back pressure curve.

Several authorities (14)(19)(31)(80) have presented methods for calculating the slope of a back pressure curve from a single flow test. Most of these methods incorporate some form of empirical relationship which attempts to account for the extreme effect that the immediate area of the well bore has on the slope of the curve. One example of such a method is that developed by Smith (80). The method accounts for the deviation of gas well behavior from laminar flow, by combining the theory of flow with a procedure for measuring and calculating well bore effects for individual wells. Smith concluded that the slope of a back pressure curve can be described by the equation for the laminar transient flow of gas in a radial system, modified to include a completion or "skin" factor and a "rate of flow" factor. The rate of flow factor accounts for the energy loss associated with the turbulence in the region of the well bore, and along with the skin factor is determined from a pressure build-up test. In cases where the slope of a back pressure curve cannot be directly measured, and where it is impossible to accurately estimate the effective permeability or well radius, and where serious deviations from homogeneity are expected, a method such as that proposed by Smith, is recommended for estimating the

slope.

Establishing a Back Pressure Curve from Gas Properties and Reservoir Characteristics

Study of equation (C-15) results in the conclusion that theoretically the pressure drop associated with any chosen flow rate can be calculated for a given reservoir, provided certain properties of the flowing gas and the reservoir can be estimated. Tek, Coats, and Katz (88) have applied this approach to develop a graphical representation of the theoretical back pressure relationship for a gas of certain gravity, as a function of the flow rate and the pay, as well as the permeability and the discovery pressure of the reservoir. This graphical representation is presented as Figure 5-2 for a 0.6 gravity gas and is useful for estimating the potential of a well to produce when no flow test data is available. The problems and limitations associated with this theoretical calculation are essentially the same as those encountered in calculating the slope from a single point test. It should be emphasized, that this theoretical computation assumes reservoir homogeneity and a circular well bore.

The following is a recommended procedure for calculating a back pressure curve by this method.

1. Estimate the average viscosity and compressibility factor on the basis of the static reservoir pressure, the approximate drawdown at which the well is normally produced, the gas gravity and the bottom hole temperature.

2. Estimate the effective permeability, pay thickness, well bore radius and radius of drainage from core analyses, logs, build-up or drawdown data if available, and from the spacing of wells in the area.

3. Calculate the turbulence factor, β, from equation (C-16).

$$\beta = \frac{4.11 \times 10^{10}}{k^{4/3}} \qquad \text{(C-16)}$$

where

k = permeability, md.

4. Assume a reasonable flowing sandface pressure, P_s.

5. From equation (D-16) and an average pressure, calculate the theoretical time to adequate stabilization. The average pressure may be taken as

$$\frac{P_f + P_s}{2}$$

where P_f and P_s are not too different but is better approximated by the

$$\sqrt{\frac{P_f^2 + P_s^2}{2}}$$

in other cases.

6. Calculate the dimensionless time corresponding to the lesser of the time to stabilization or 15 days, from equation (C-32) and the assumed sandface pressure.

7. Determine from Figure C-2, a dimensionless drawdown, P_t, corresponding to the time of stabilization.

8. Calculate the dimensionless flow rate, q_D, from equation (C-34).

9. Calculate the flow rate, Q, corresponding to 'q_D' from equation (C-33).

10. Calculate the sandface pressure, P_s, from equation (C-15). If this pressure differs significantly from the assumed pressure, steps 4 to 10 are repeated, using the calculated P_s in step 10 as the assumed P_s in step 4. (Steps 4 to 9 inclusive, calculate a stabilized flow rate corresponding to a reasonable sandface pressure. These steps may be omitted, a stabilized flow rate assumed, and a sandface pressure calculated as

in step 10. If the calculated pressure is not a reasonable one, another flow rate is assumed and step 10 is repeated until the resulting sandface pressure is reasonable.)

11. Plot on log coordinates, the resulting $(P_f^2 - P_s^2)$ against the 'Q' calculated in step 9. Determine the turbulence contribution factor, F_t, at this flow rate, and adjust the point to reflect only laminar flow. A line representing laminar flow conditions (slope equal to unity) is then drawn through the point.

12. Adjust the line for turbulence by the method previously presented. The resulting curve should represent the capability of the well to produce under stabilized conditions.

The following example illustrates the recommended procedures for determining the proper slope of a back pressure curve and for positioning the stabilized curve. It also illustrates the mechanics of the calculations involved in constructing a stabilized back pressure curve from gas and reservoir properties.

<u>Example 5-1</u>

A well is flowed at a rate of 5.0 MMcfd for three hours. The sandface pressure after this time is 2890 psia. The well is drilled on one section spacing and the following reservoir and flow characteristics are known.

P_f = 3030 psia (from previous pressure build-up).

G = 0.78

μ_a = 0.02 (at estimated average reservoir conditions).

k = 14 md (from previous pressure build-up).

ϕ = 0.07

r_s = 0.3 ft.

h = 45 ft.

T_a = 630°R.

Z_a = 0.81 (at estimated average reservoir conditions).

Construct a stabilized back pressure plot of correct slope.

Solution

At Q = 5.0 MMcfd

$P_f^2 - P_s^2$ = 829,000

This point is plotted in the conventional manner (see Figure 5-1).

The time to which the line is positioned to reflect adequate stabilization may be calculated from equation (D-16).

$$t_s = 1000 \frac{r_f^2 \mu_a \phi}{k\ P_f} = 1000 \frac{(2640)^2(0.02)(0.07)}{(14)(3030)} = 230 \text{ hours}$$

The dimensionless time corresponding to three hours is determined from equation (C-32).

$$t_D = \frac{2.634 \times 10^{-4}\ (14)(2960)(3)}{(0.02)(0.07)(0.3)^2} = 260{,}000$$

From Figure C-2

$$P_t = 6.60$$

and from equation (C-34)

$$q_D = \left(\frac{3030^2 - 2890^2}{3030^2}\right) \frac{1}{6.60} = 0.0137$$

Assume a sandface pressure of 2800 psia after 230 hours.

$$P_a = \frac{3030 + 2800}{2} = 2915 \text{ psia}$$

From equation (C-32)

$$t_D = \frac{2.634 \times 10^{-4} (14)(2915)(230)}{(0.02)(0.07)(0.3)^2} = 1.96 \times 10^7$$

From Figure C-2

$$P_t = 8.75$$

Substituting in equation (C-34)

$$\frac{3030^2 - P_s^{\,2}}{3030^2} = 0.0137\ (8.75)$$

$$P_s = 2843 \text{ psia}$$

So after 230 hours, and at a flow rate of 5 MMcfd, $P_f^{\,2} - P_s^{\,2} = 1,100,000$.

The apparent radius of drainage may be determined from the relationship developed by Janicek and Katz (45).

$$r_a = 0.749(2640) = 1977 \text{ ft.}$$

F_t is then evaluated from equation (C-23).

$$F_t = 1 + \frac{0.09101\ (0.78)\ Q}{(0.02)(14)^{1/3}(0.3)(45)\ln(1977/0.3)} = 1 + 0.0124\ Q$$

At 5 MMcfd, the turbulence contribution factor is

$$F_t = 1 + 0.0124\ (5.0) = 1.062$$

So the value of $P_f^{\,2} - P_s^{\,2}$, under stabilized laminar flow conditions would be

$$P_f^{\,2} - P_s^{\,2} = \frac{1,100,000}{1.062} = 1,036,000$$

This point is plotted on Figure 5-1, and a line of slope unity is drawn through it.

The value of $P_f^{\,2} - P_s^{\,2}$, for various flow rates under laminar

conditions is then multiplied by the turbulence contribution factor to correct for turbulence. If we assume that the well in question will normally be produced over the range 2 to 20 MMcfd, the turbulence correction is calculated accordingly.

Q-MMcfd	F_t	$(P_f^2 - P_s^2) \times 10^{-6}$ (from laminar plot)	$(P_f^2 - P_s^2) \times 10^{-6}$ (adjusted)
2	1.025	0.41	0.42
5	1.062	1.04	1.10
10	1.124	2.11	2.37
15	1.186	3.20	3.80
20	1.248	4.25	5.30

The adjusted values of $P_f^2 - P_s^2$ are then plotted on Figure 5-1 and the best fit straight line is drawn through the points. The resulting value of 'n' is 0.93. This line represents the stabilized back pressure line.

Had the flow point of 5 MMcfd at a pressure of 2890 psia not been known, it could have been estimated, or the stabilized point could be estimated as follows.

The turbulence factor, β, is calculated from equation (C-16).

$$\beta = \frac{4.11 \times 10^{10}}{(14)^{4/3}} = 12.2 \times 10^8$$

The value of $(P_f^2 - P_s^2)$ is then calculated for the condition of stabilization, from equation (C-15).

$$P_f^2 - P_s^2 = \frac{1.418 \times 10^6 \,(0.02)(630)(0.81)(5.0)\,\ln(1977/0.3)}{(14)(45)} +$$

$$\frac{3.14 \times 10^{-6}\,(12.2 \times 10^8)(0.78)(0.81)(630)(5^2)}{(45^2)}\left(\frac{1}{0.3} - \frac{1}{1977}\right)$$

$$= 1{,}073{,}000$$

Knowing the formation pressure is 3030 psia,

$$P_s = 2847 \text{ psia}$$

This compares with the stabilized sandface pressure of 2843 psia which was calculated from the pressure measured at three hours.

All of the methods presented or referred to in this section are approximations, consequently the resulting estimation of flow behavior is of limited use. A reasonable application of these methods, however, could be the interpretation of single point tests conducted periodically throughout the life of a well, provided, they are spaced appropriately with regard to multi-point tests. Another important use might be interpreting tests where the results of a multi-point test appear to be erroneous. In these situations, calculations may lead to a more representative back pressure curve than that resulting from a series of flow rates. In any case, due largely to lack of reservoir homogeneity, theory of flow in a reservoir has not yet advanced to the point where methods of calculation can replace actual flow tests.

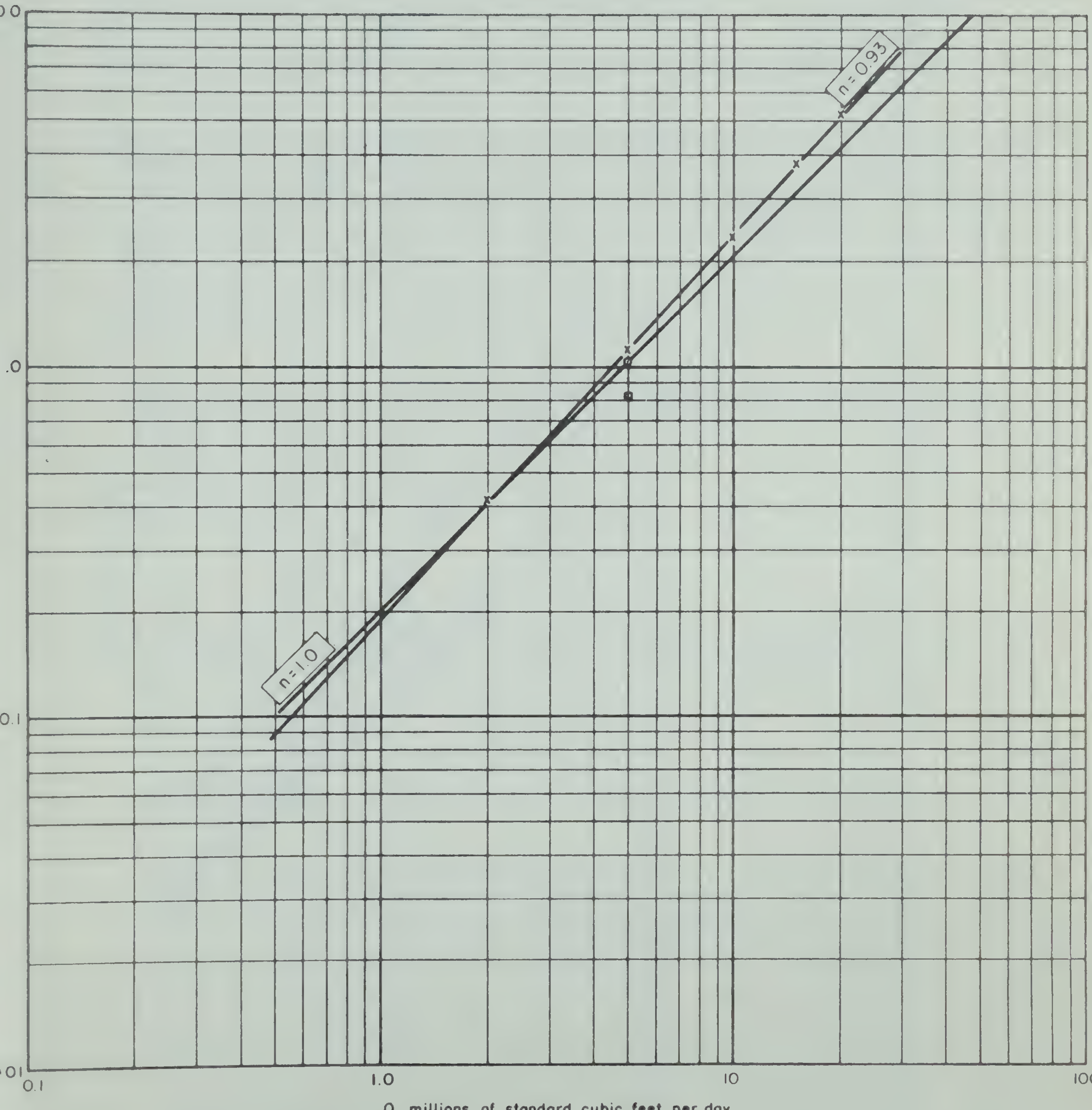


FIGURE 5-1 — CONSTRUCTION OF A STABILIZED BACK PRESSURE CURVE FROM A SINGLE FLOW POINT.

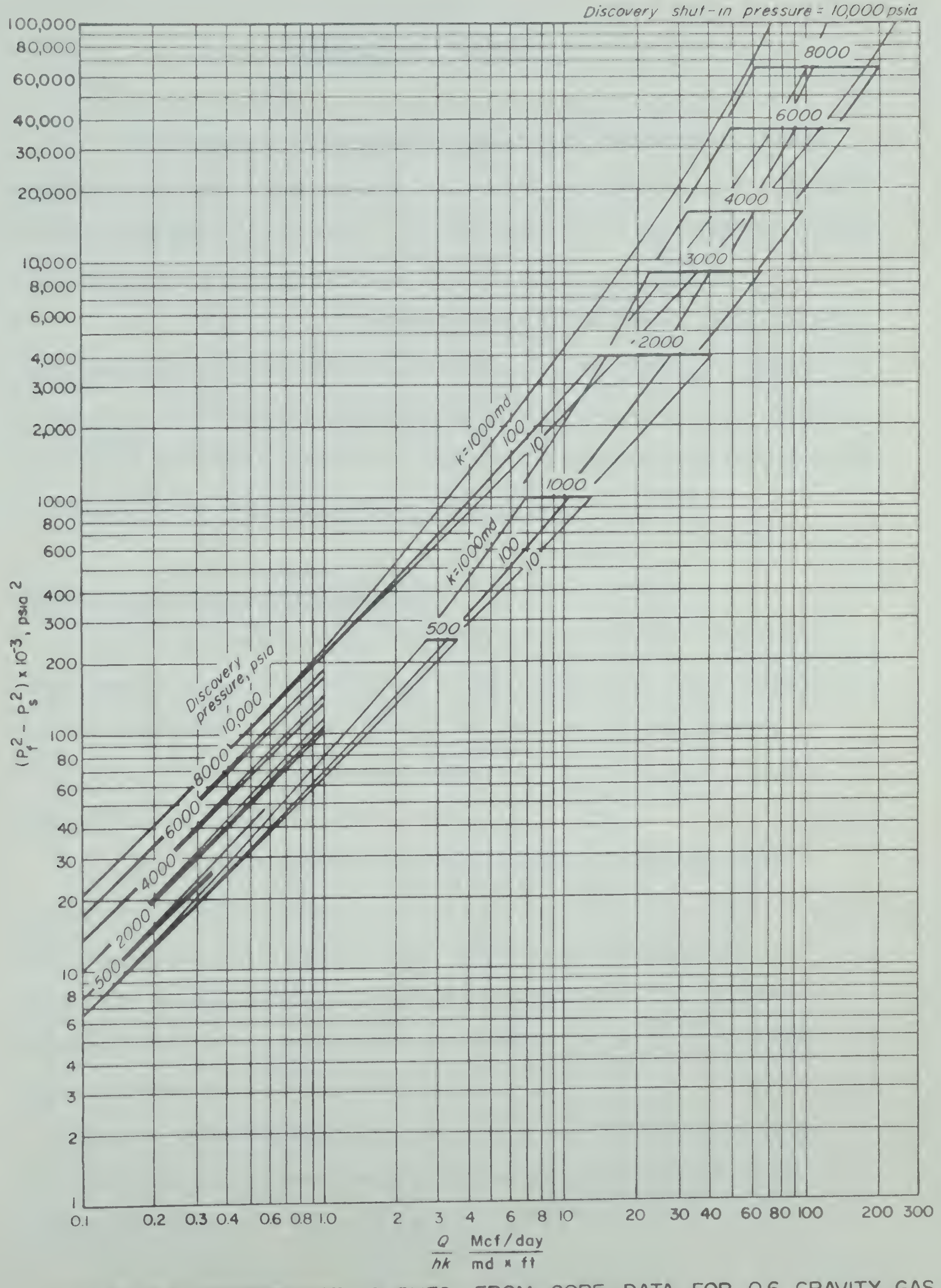


FIGURE 5-2-PERFORMANCE CURVES FROM CORE DATA FOR 0.6 GRAVITY GAS

(Reproduced from Tek Coats and Katz (88).)

APPENDIX A

NATURAL GAS PROPERTIES

This appendix is intended to familiarize the reader with those physical properties of natural gas that are of importance in the evaluation of the performance of gas wells. It also presents a summary of the formulae and commonly used graphs and tables related to these properties.

The properties of a natural gas may be determined either directly from laboratory tests or by calculation from the known chemical composition of the gas. In the latter case the calculations are based on the properties of the individual component of the gas and upon physical laws relating the properties of the components to those of the gas mixture.

Properties of Constituent Components

The relevant physical properties of the important constituent components of natural gas are listed in Table A-1. This table presents the recognized values of the critical temperature, critical pressure, molecular weight, specific gravity, vaporizing volume ratio and heating value of these components.

The critical pressure and the critical temperature are the pressure and temperature under which the gaseous and liquid states become indistinguishable. For a single component system, the critical temperature is the temperature above which a gas cannot be liquified by the application of pressure alone. The critical pressure is the vapor pressure at the critical temperature.

The molecular weight is the sum of the atomic weights of the atoms in a molecule of the substance, the weight of an atom of oxygen being taken as 16.

The specific gravity of a gaseous substance is the ratio of the weight of a volume of the gas to that of the same volume of pure dry air, both at a standard pressure and temperature. The specific gravity may be determined by direct measurement, but, from the above definition, it may readily be demonstrated that the specific gravity of an ideal gas is the ratio of the molecular weight of the gas to the molecular weight of pure dry air which is 28.966. For practical purposes, it may be considered that this relationship also applies to real gases.

The vaporizing volume ratio is the ratio of a volume of gas at a standard pressure and temperature to the volume that it would occupy in the liquid state at the same pressure and temperature.

The gross (or higher) heating value of a gas is the total heat liberated when a unit volume of the gas is burned under specified conditions. The net (or lower) heating value is the total heat liberated, less the latent heat of vaporization of the water formed in the combustion. This property is usually expressed in British Thermal Units of heat per cubic foot of gas at standard conditions.

The Gas Law

In the case of gases a relationship between pressure, temperature and specific volume exists. This relationship is commonly known as the Gas Law and may be written as follows.

$$Pv = \frac{ZRT}{144\ M} \qquad \text{(A-1)}$$

where

P = pressure under which the gas exists, psia.

v = specific volume of the gas, cubic feet per pound.

Z = compressibility factor for the gas.

T = absolute temperature, degrees Rankine (degrees Rankine = degrees Fahrenheit + 460).

R = Gas Law constant, 1545 foot pounds per pound mole degrees Rankine.

M = molecular weight of the gas, pounds per pound mole.

Equation (A-1) can be rearranged in a more convenient form by introducing the relationship

$$M = 28.966\ G \tag{A-2}$$

where

G = specific gravity of the gas.

then

$$\frac{1}{v} = \rho = \frac{144\ GP}{53.34\ TZ} \tag{A-3}$$

where

ρ = density of the gas, lbs. - mass per cubic foot.

Compressibility of Natural Gases

The compressibility factor, Z, is defined by equation (A-1) and in fact, is the factor which validates the equation. It is best determined experimentally for any gas and is found to be a function of temperature, pressure and composition.

According to the theorem of corresponding states, two substances should have similar properties when at "corresponding states". The corresponding state is taken as the state existing at corresponding

fractions of the critical temperature and the critical pressure. The ratio of the actual temperature to the critical temperature is called the reduced temperature and the ratio of the actual pressure to the critical pressure is called the reduced pressure. Thus, one would expect that at the same reduced temperature and pressure, the compressibility factors of two hydrocarbons would be the same.

For mixtures such as natural gas the molal average critical temperature and pressure may be used in place of the true critical temperature and critical pressure for the purpose of establishing corresponding states. These molal average properties are called the pseudo critical temperature and the pseudo critical pressure and may be calculated from the composition of the gas as illustrated in Examples A-1 and A-2.

Matthews, et al (57) found a reasonably systematic relationship between the pseudo critical properties and the gas gravity for a wide range of gases containing less than about three per cent non-hydrocarbons. Further work (10) has led to the development of the correlation presented in Figure A-1. The commonly used upper curve of this figure is reproduced in tabular form in the Interstate Oil Compact Commission (103) and the Kansas State Corporation Commission (104) test manuals. Figure A-1 is useful for estimating the pseudo critical properties in cases where the gas analysis is not available.

Standing and Katz (82) have developed the most widely accepted correlation of the compressibility factor in terms of pseudo reduced temperature and pseudo reduced pressure for natural gases. This correlation is presented in graphical form in Figure A-2 and appears in tabular form in the previously mentioned back pressure testing manuals. The Standing and Katz correlation is restricted to natural gases containing small amounts (say less than two per cent in total) of carbon dioxide and hydrogen sulphide.

Recent work at the University of Alberta (73) sponsored by the Alberta Oil and Gas Conservation Board, has resulted in a method for estimating the effect of the acid gas constituents on the compressibility factor. The method involves the use of the compressibility factor correction ratios presented in Figures A-3 to A-8, inclusive. The procedure is as follows:

1. Calculate the pseudo reduced properties of the whole gas.

2. Determine the approximate compressibility factor from Figure A-2.

3. Determine the compressibility factor correction ratio from Figures A-3 to A-8.

4. Calculate the final compressibility factor by dividing the quantity determined in item 2 by that determined in item 3.

The compressibility factors resulting from the correlations in Figures A-3 to A-8 have been tested against actual experimental measurements for a number of sour natural gases, and are believed reliable to within one per cent for sour gases containing less than five mol per cent pentanes plus.

Examples A-1 and A-2 illustrate the use of the above mentioned methods for determining the compressibility factor for a sweet and a sour natural gas.

<u>Example A-1</u>

(a) Compute the compressibility factor at a pressure of 1500 psia and a temperature of 100°F of a sweet natural gas having the composition shown below.

For convenience, the following symbols will be used in the solution of this problem and, in fact, throughout the manual.

P_c = critical pressure, psia.

T_c = critical temperature, ^oR.

P_r = reduced pressure or the ratio of the actual pressure to critical pressure.

T_r = reduced temperature or the ratio of the actual temperature to the critical temperature.

The above terminology will apply whether the property referred to is a true property for a pure component of a natural gas or a pseudo property for a mixture of these components.

Solution

The pseudo critical properties of the natural gas are calculated by summing the products of the mol fraction times the critical properties for each component.

COMPONENT	MOL. WT.	MOL %	M.W. x MOL FRACTION	CRITICAL PRESSURE P_c	CRITICAL TEMPERATURE T_c	MOL FRACTION x P_c	MOL FRACTION x T_c
N_2	28.016	2.25	0.6304	492.3	226.9	11.08	5.11
CH_4	16.042	86.89	13.9389	673.1	343.2	584.86	298.21
C_2H_6	30.068	6.20	1.8642	708.3	549.8	43.91	34.09
C_3H_8	44.094	2.56	1.1288	617.2	666.0	15.80	17.05
iC_4H_{10}	58.120	0.49	0.2848	529.0	734.7	2.59	3.60
nC_4H_{10}	58.120	0.74	0.4301	551.1	765.3	4.08	5.66
iC_5H_{12}	72.146	0.24	0.1732	483.5	829.7	1.16	1.99
nC_5H_{12}	72.146	0.18	0.1299	489.4	845.6	0.88	1.52
*C_6H_{14}+	100.198	0.45	0.4509	396.8	972.3	1.79	4.38
			19.0312			Pseudo P_c = 666	Pseudo T_c = 372

* There is a problem in knowing what critical properties should be used for the residual mixture of components. Kay (50) devised a method for obtaining the pseudo critical properties of liquids through a relationship to the molecular weight and specific gravity of the mixture. However, it is convenient (and normally quite accurate) to use the critical properties of the next higher component. In this example the properties of heptanes have been used for the hexanes plus fraction.

$$P_r = \frac{1500}{666} = 2.25$$

$$T_r = \frac{560}{372} = 1.51$$

From Figure A-1, at $P_r = 2.25$ and $T_r = 1.51$, the compressibility factor, $Z = 0.812$.

(b) Assuming only the specific gravity of the gas were known, calculate the compressibility using the Brown et al correlation of pseudo critical properties and specific gravity.

Solution

The specific gravity of the gas $= \frac{19.0312}{28.966} = 0.657$

From Figure A-1 for

$G = 0.657$

$P_c = 669$ psia

$T_c = 376^{\circ}R$

$$P_r = \frac{1500}{669} = 2.24$$

$$T_r = \frac{560}{376} = 1.49$$

$Z = 0.805$ (Figure A-2)

The excellent agreement indicates the relative accuracy frequently obtained through the use of Figure A-1 for a normal sweet gas, but should not be taken as indicative of the accuracy that will always be attained.

Example A-2

Compute the compressibility factor at a pressure of 1500 psia and a temperature of 100°F of a sour natural gas with the composition shown below.

Solution

Since the gas contains greater than three per cent non-hydrocarbons, Figure A-1 is not suitable to estimate the pseudo critical properties. Consequently, the molal average of these properties must be calculated.

COMPONENT	MOL. WT.	MOL %	M.W. x MOL FRACTION	CRITICAL PRESSURE P_c	CRITICAL TEMPERATURE T_c	MOL FRACTION x P_c	MOL FRACTION x T_c
H_2S	34.076	14.38	4.9001	1306.5	672.4	187.87	96.69
CO_2	44.010	0.30	0.1320	1072.8	547.7	3.22	1.64
N_2	28.016	0.46	0.1289	492.3	226.9	2.26	1.04
CH_4	16.042	84.14	13.4977	673.1	343.2	566.35	288.77
C_2H_6	30.068	0.59	0.1774	708.3	549.8	4.18	3.24
C_3H_8	44.094	0.08	0.0353	617.2	666.0	0.49	0.53
iC_4H_{10}	58.120	0.03	0.0174	529.0	734.7	0.16	0.22
nC_4H_{10}	58.120	0.02	0.0116	551.1	765.3	0.11	0.15
C_5H_{12}+		Trace					
			18.9004			Pseudo P_c = 765	Pseudo T_c = 392

$$P_r = \frac{1500}{765} = 1.96$$

$$T_r = \frac{560}{392} = 1.43$$

From Figure A-2, the approximate compressibility factor is determined.

$$Z = 0.790$$

From Figures A-3 and A-4 the compressibility correction ratio = 0.960

$$\text{Corrected } Z = \frac{Z \text{ (Approximate)}}{\text{Correction Factor}} = \frac{0.790}{0.960} = 0.823$$

It can be seen from the preceeding example that for sour natural gases appreciable errors are involved if only the Standing and Katz correlation (Figure A-2) is used for calculating the compressibility factor.

Specific Gravity of Condensate Well Effluents

Many gas wells produce an effluent which separates into a gas and a condensate at the wellhead operating pressure and temperature. Frequently the separated liquid results from retrograde condensation and the fluid in the well bore itself is completely in the gaseous phase. The specific gravity of such a well bore gas may be calculated, assuming additive volumes, from the specific gravity of the gas separated at the wellhead, the gas condensate ratio, and the specific gravity and the vaporizing volume ratio of the condensate at separator conditions. The equation relating these factors is

$$G_m = \frac{G_s + \dfrac{4608\ G_c}{R_c}}{1 + V_c/R_c} \qquad \text{(A-4)}$$

where

G_m = specific gravity of the single phase gas in the well bore (Air = 1).

G_s = specific gravity of the separator gas (Air = 1).

G_c = specific gravity of the separator condensate (Water = 1).

R_c = gas condensate ratio, cubic feet per barrel (14.65 psi and 60°F).

V_c = condensate vaporizing volume ratio, cubic feet per barrel (14.65 psi and 60°F).

Condensate Vaporizing Volume Ratio

The condensate vaporizing volume ratio varies with the composition of the condensate which in turn is reflected by its specific gravity. Figure A-9 presents the relationship between the vaporizing volume and the API gravity for pure pariffinic hydrocarbons. (For convenience in converting API gravities to liquid specific gravities, an abridged conversion table is presented as Table A-2). This relationship may be used for condensate of normal composition without appreciable error.

Equation (A-4) makes no allowance in the calculation of specific gravity for water vapor that may be produced with the gas. Experiments and calculations both indicate that in the absence of liquid water in the flow stream, even under conditions of maximum water vapor content, the error resulting from this assumption is negligible.

The following example demonstrates the use of the equation.

Example A-3

Given the following information with respect to separator products, calculate the specific gravity of the single phase gas in the well bore.

G_s = 0.70

G_c = 56° API

R_c = 15,000 cu.ft./bbl. at 14.65 psi and 60°F.

Solution

G_c = 0.755 (Table A-2)

V_c = 575 cu.ft./bbl. at 14.65 psi and 60°F. (Figure A-9)

Substituting in equation (A-4)

$$G_m = \frac{0.70 + \frac{4608(0.755)}{15,000}}{1 + \frac{575}{15,000}}$$

$$G_m = 0.898$$

Viscosity

The viscosity of a pure gas depends upon both pressure and temperature, and in the case of a natural gas also depends upon the composition of the mixture. Carr, Kobayashi, and Burrows (13) have developed correlations for determining the viscosity of a natural gas. These correlations appear as Figures A-10, A-11 and A-12. Figure A-10

is a plot of the viscosity at one atmosphere pressure as a function of the molecular weight and the temperature of the mixture. Figures A-11 and A-12 give the viscosity ratio as a function of the pseudo reduced temperature and the pseudo reduced pressure. The viscosity ratio is the ratio of the viscosity at a given pressure to the viscosity at one atmosphere pressure.

Use of the above mentioned correlations give reliable results for gases that contain no nitrogen, carbon dioxide or hydrogen sulphide. Corrections for the presence of these constituents may be made through the use of the insert plots of Figure A-10.

The following example illustrates the use of the above mentioned correlations.

<u>Example A-4</u>

Calculate the viscosity of the natural gas of Example A-2 at $140^{\circ}F$ and 2000 psia.

The following symbols and definitions are for convenience and will be used throughout the manual.

μ = viscosity, centipoise (at any given pressure and temperature condition).

$\mu 1$ = viscosity, centipoise (at a pressure of one atmosphere and any temperature condition).

$\mu/\mu 1$ = viscosity ratio as determined from Figures A-11 and A-12.

Solution

From Figure A-10 for a gas with a molecular weight of 18.9 and at a temperature of 140°F

$\mu 1 = 0.0116$

Correction added to viscosity for 0.652 gravity gas with 14.38% H_2S

$= 0.0003$ (Figure A-10)

Corrected $\mu 1 = 0.0116 + 0.0003 = 0.0119$ cp

$P_c = 765$ psia

$T_c = 393^\circ R$

$$P_r = \frac{2000}{765} = 2.61$$

$$T_r = \frac{600}{392} = 1.53$$

From Figure A-11 for $P_r = 2.61$ and $T_r = 1.53$

$\mu/\mu 1 = 1.37$

$\mu = 1.37\ (0.0119) = 0.0163$ cp.

TABLE A-1

PHYSICAL PROPERTIES OF IMPORTANT COMPONENTS OF NATURAL GAS

Component	P_c (psia)	T_c (^oR)	Molecular Weight	Ft^3 Gas/Imp Gal @ 60^oF and 14.65 psia	Gross Heating Value, Btu/ft^3 @ 60^oF and 14.65 psia (dry)
H_2S	1306.5	672.4	34.076	88.22	678
CO_2	1072.8	547.7	44.010	70.18	-
N_2	492.3	226.9	28.016	-	-
He	33.1	9.2	4.003	-	-
CH_4	673.1	343.2	16.042	71.02	1007
C_2H_6	708.3	549.8	30.068	47.25	1763
C_3H_8	617.2	666.0	44.094	43.76	2510
iC_4H_{10}	529.0	734.7	58.120	36.83	3246
nC_4H_{10}	551.1	765.3	58.120	38.22	3254
Butanes*	543.7	755.1	58.120	37.76	3251
iC_5H_{12}	483.5	829.7	72.146	32.98	3988
nC_5H_{12}	489.4	845.6	72.146	33.32	3996
Pentanes**	486.2	836.9	72.146	33.12	3992
nC_6H_{14}	440.0	914.2	86.172	29.35	4741
nC_7H_{16}	396.8	972.3	100.198	26.16	5486
nC_8H_{18}	361.5	1024.9	114.224	23.57	6230
C_9H_{20}	345.0	1073.0	128.250	21.434	6975
$C_{10}H_{22}$	320.0	1115.0	142.276	19.652	7720

* Butanes = 1/3 iC_4 and 2/3 nC_4

** Pentanes = 55% iC_5 and 45% nC_5

TABLE A-2

CONVERSION OF API GRAVITY TO LIQUID SPECIFIC GRAVITY AT 60°F.

$$\text{Specific Gravity} = \frac{141.5}{131.5 + {}^{\circ}\text{API}}$$

°API	Specific Gravity	°API	Specific Gravity	°API	Specific Gravity
0	1.076	34	.8550	68	.7093
1	1.068	35	.8498	69	.7057
2	1.060	36	.8448	70	.7022
3	1.052	37	.8398	71	.6988
4	1.044	38	.8348	72	.6953
5	1.037	39	.8299	73	.6919
6	1.029	40	.8251	74	.6886
7	1.022	41	.8203	75	.6852
8	1.014	42	.8155	76	.6819
9	1.007	43	.8109	77	.6787
10	1.000	44	.8063	78	.6754
11	.9930	45	.8017	79	.6722
12	.9861	46	.7972	80	.6690
13	.9792	47	.7927	81	.6659
14	.9725	48	.7883	82	.6628
15	.9659	49	.7839	83	.6597
16	.9593	50	.7796	84	.6566
17	.9529	51	.7753	85	.6536
18	.9465	52	.7711	86	.6506
19	.9402	53	.7669	87	.6476
20	.9340	54	.7628	88	.6446
21	.9279	55	.7587	89	.6417
22	.9218	56	.7547	90	.6388
23	.9159	57	.7507	91	.6360
24	.9100	58	.7467	92	.6331
25	.9042	59	.7428	93	.6303
26	.8984	60	.7389	94	.6275
27	.8927	61	.7351	95	.6247
28	.8871	62	.7313	96	.6220
29	.8816	63	.7275	97	.6193
30	.8762	64	.7238	98	.6166
31	.8708	65	.7201	99	.6139
32	.8654	66	.7165	100	.6112
33	.8602	67	.7128		

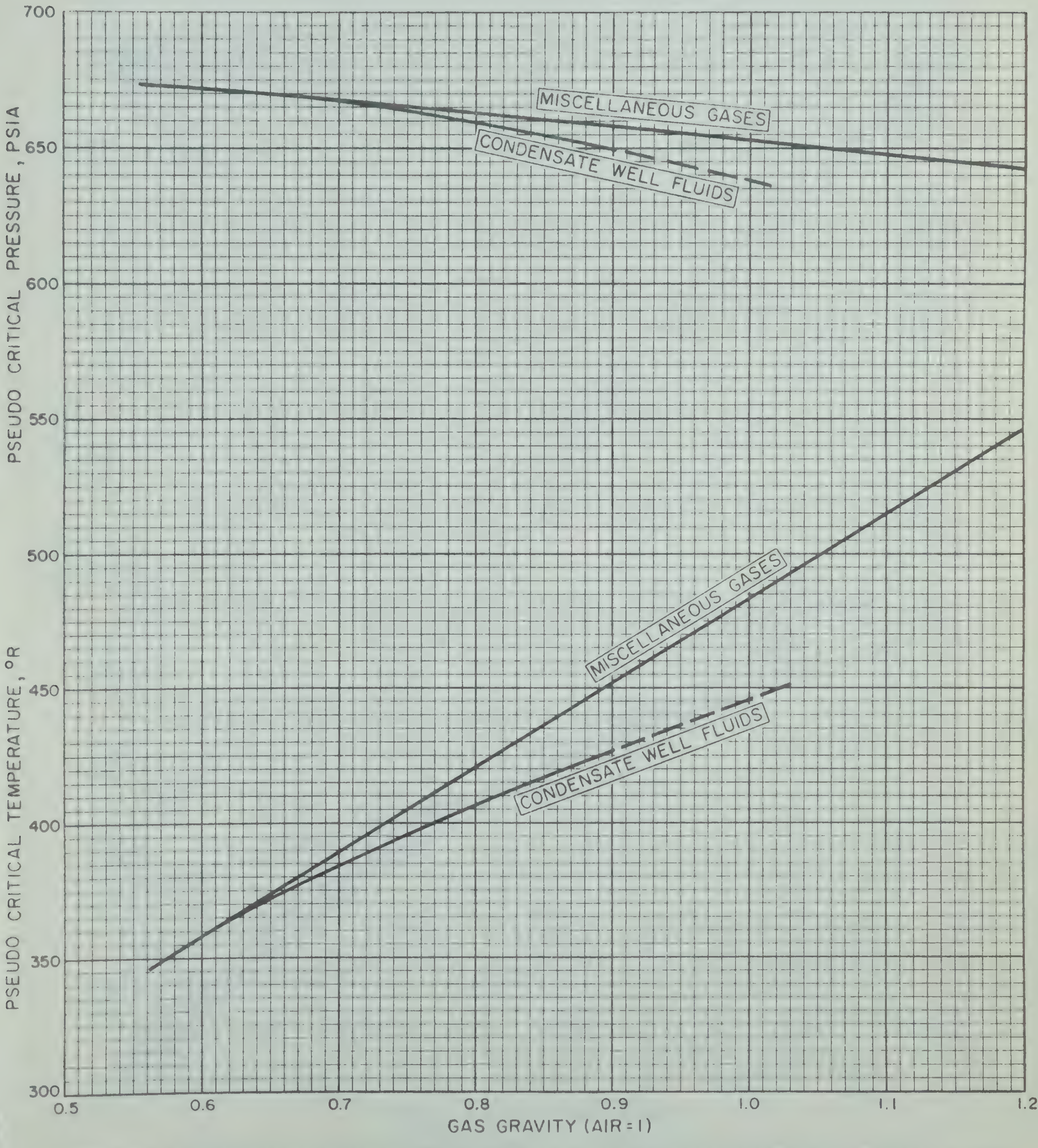


FIGURE A-1 — PSEUDO CRITICAL PROPERTIES OF MISCELLANEOUS NATURAL GASES.
(Redrawn from Brown et al, 10)

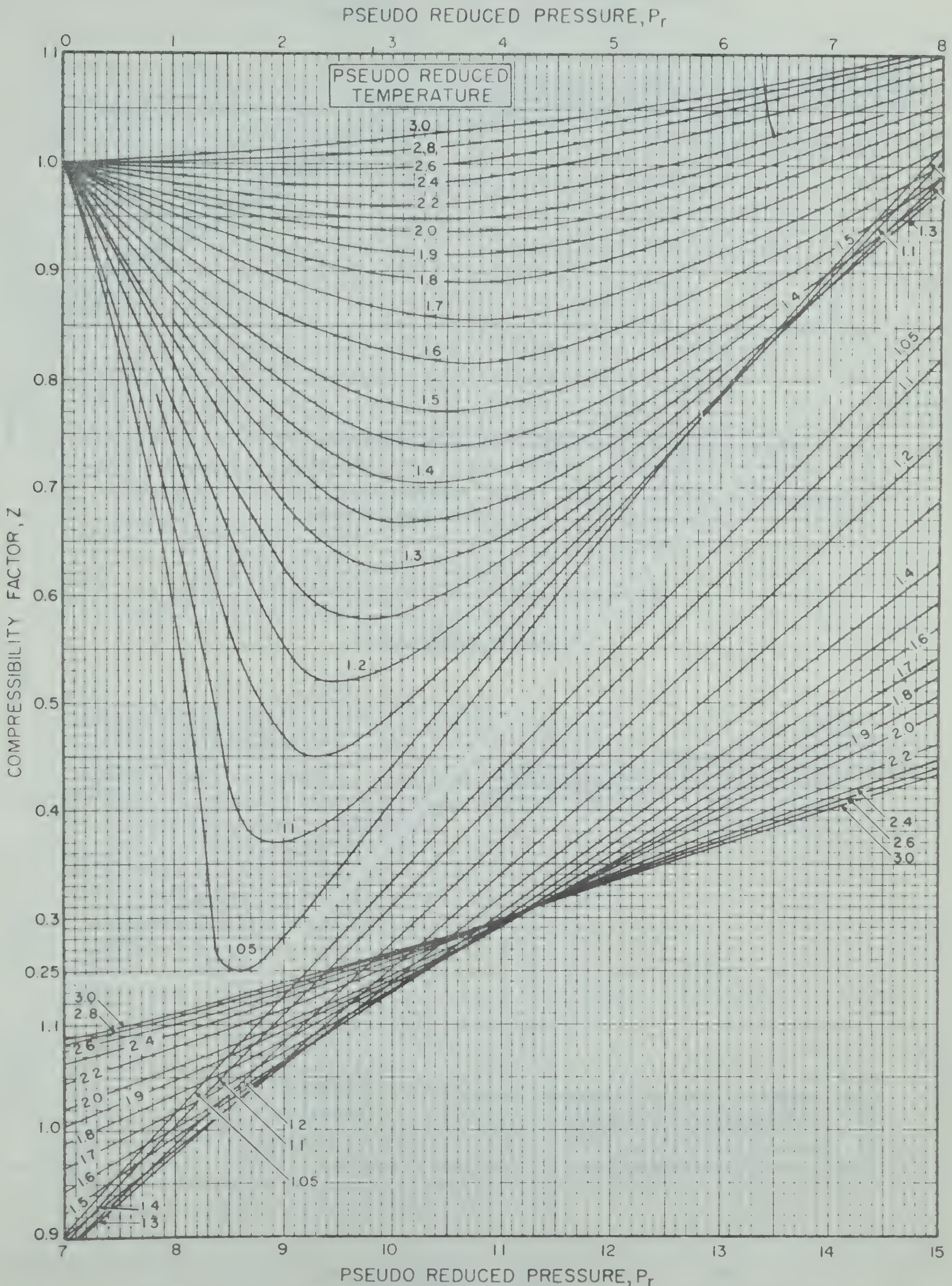
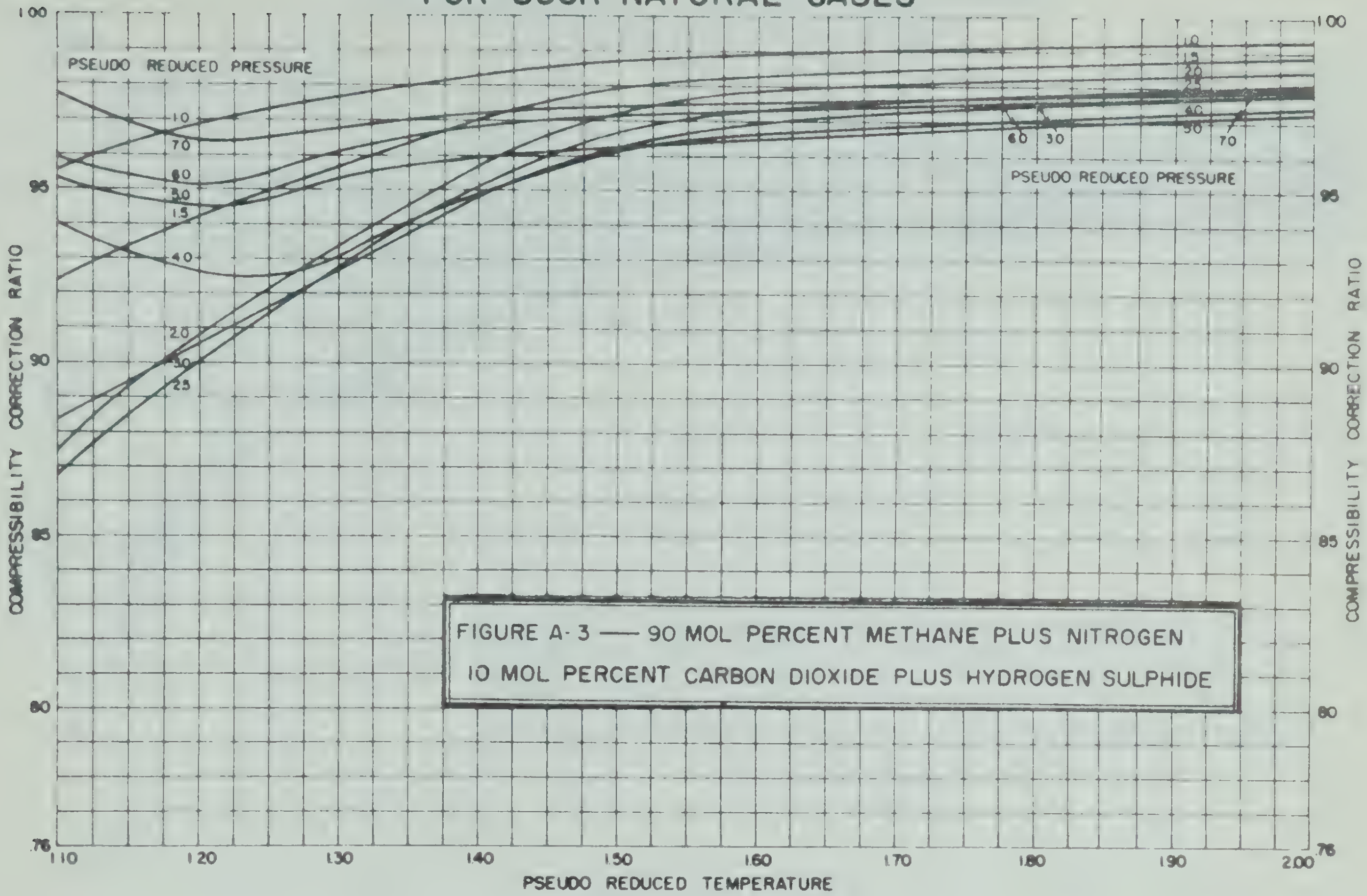
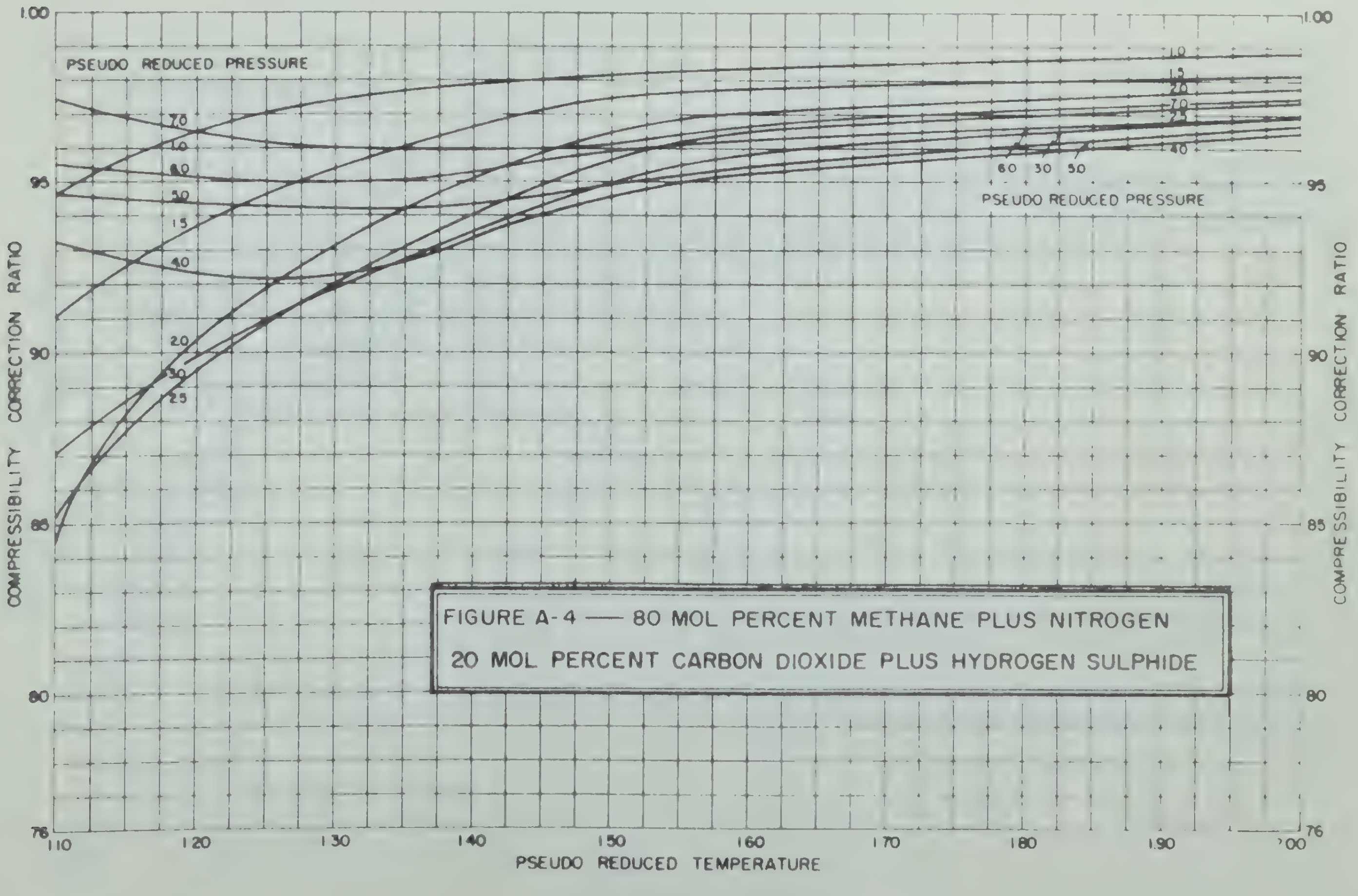


FIGURE A-2 - THE COMPRESSIBILITY FACTOR FOR NATURAL GASES.

(Redrawn from Standing and Katz, 82. Courtesy AIME.)

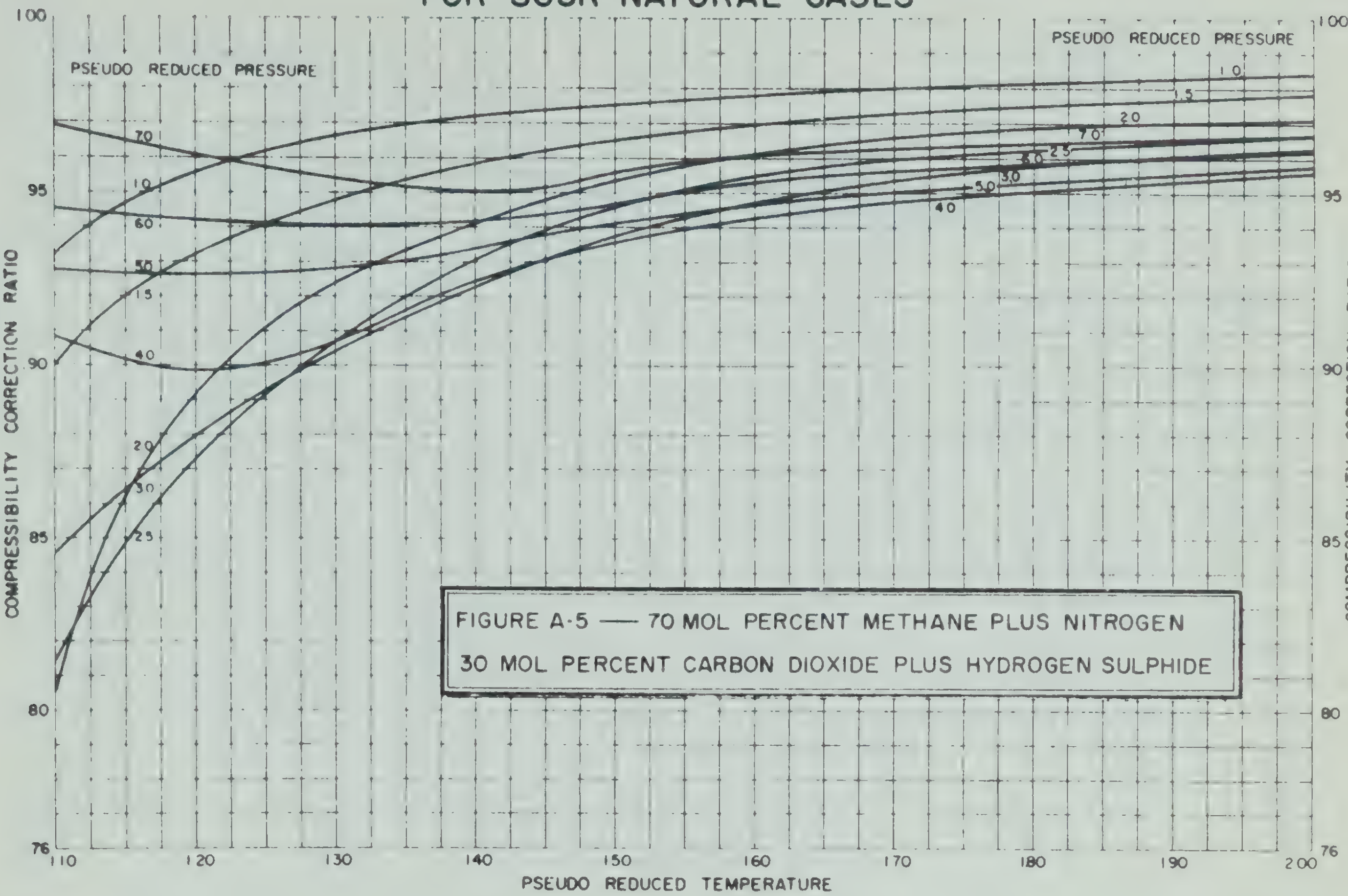
COMPRESSIBILITY CORRECTION RATIOS FOR SOUR NATURAL GASES

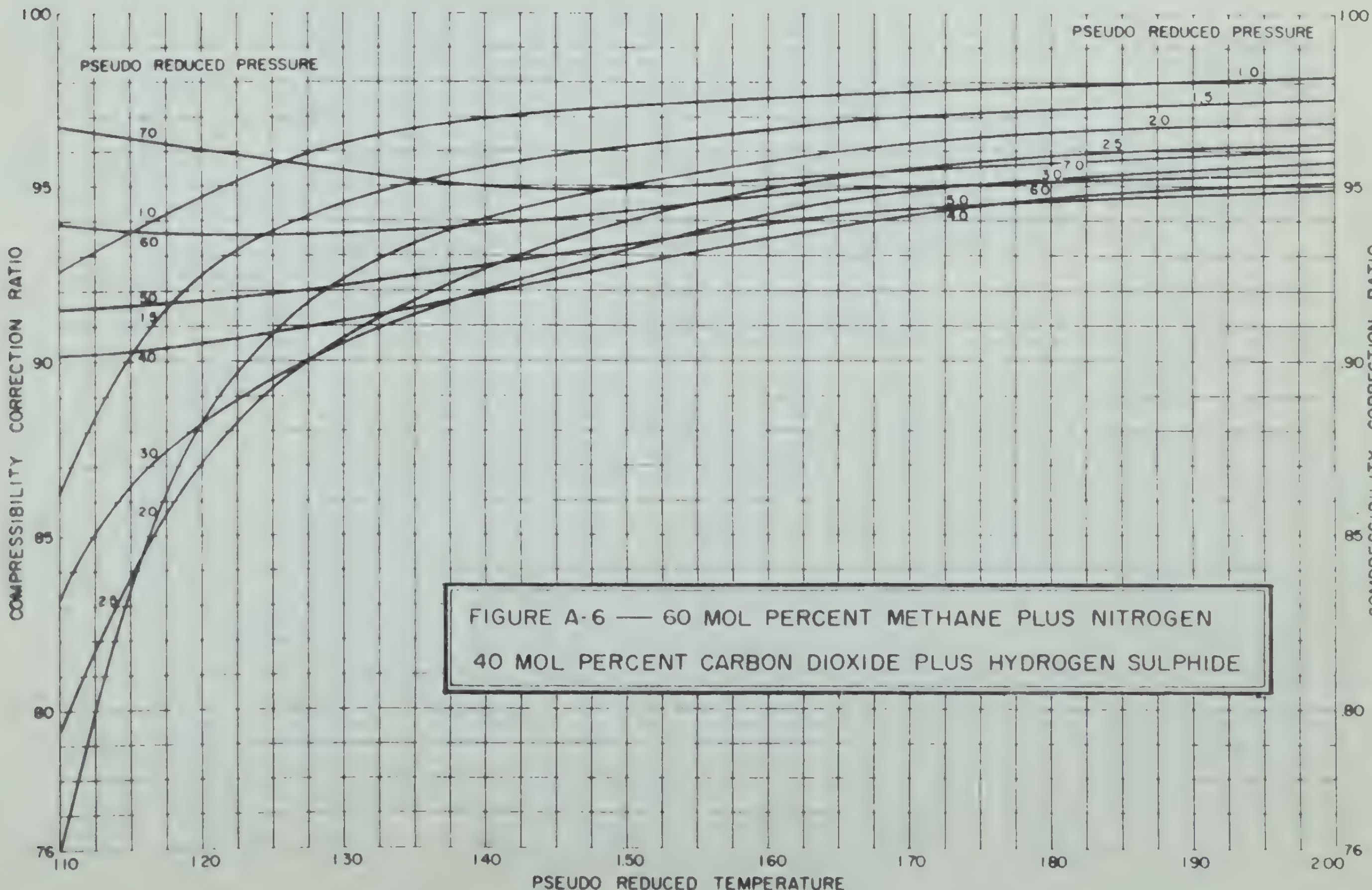




REPRODUCED FROM ROBINSON et al, (73)

COMPRESSIBILITY CORRECTION RATIOS FOR SOUR NATURAL GASES





REPRODUCED FROM ROBINSON et al,(73)

COMPRESSIBILITY CORRECTION RATIOS FOR SOUR NATURAL GASES

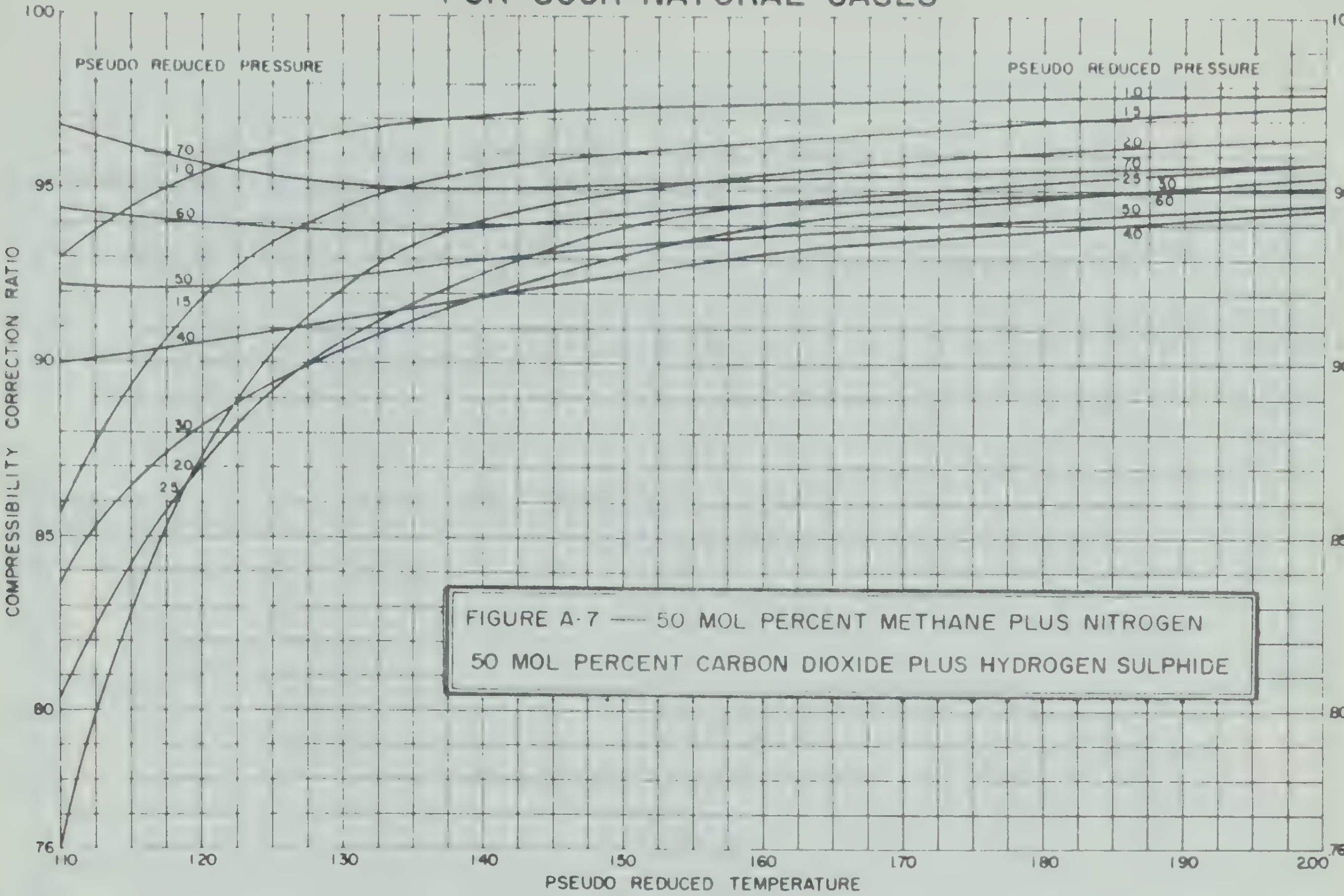


FIGURE A-7 — 50 MOL PERCENT METHANE PLUS NITROGEN
50 MOL PERCENT CARBON DIOXIDE PLUS HYDROGEN SULPHIDE

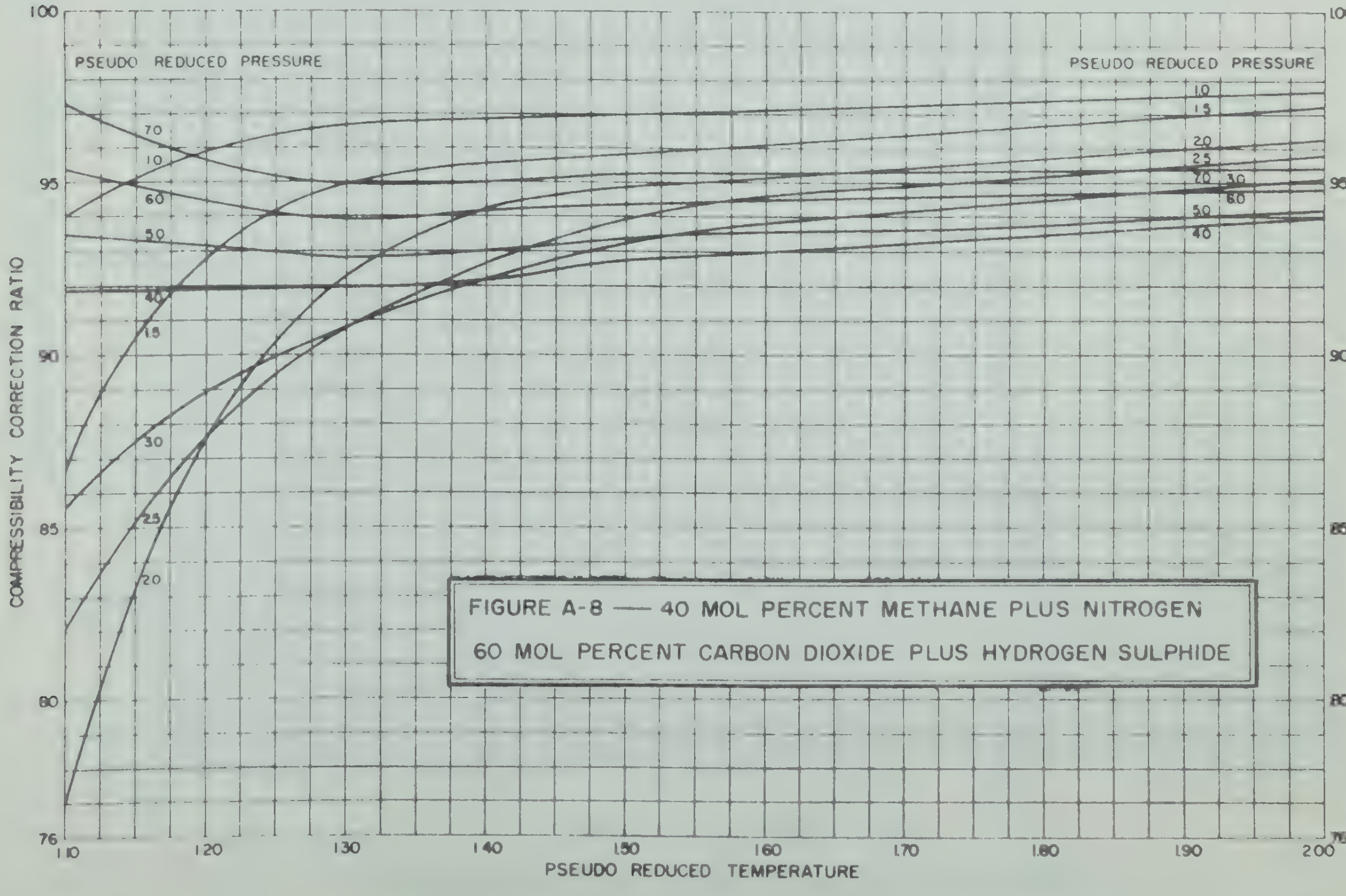


FIGURE A-8 — 40 MOL PERCENT METHANE PLUS NITROGEN
60 MOL PERCENT CARBON DIOXIDE PLUS HYDROGEN SULPHIDE

REPRODUCED FROM ROBINSON et al, (73)

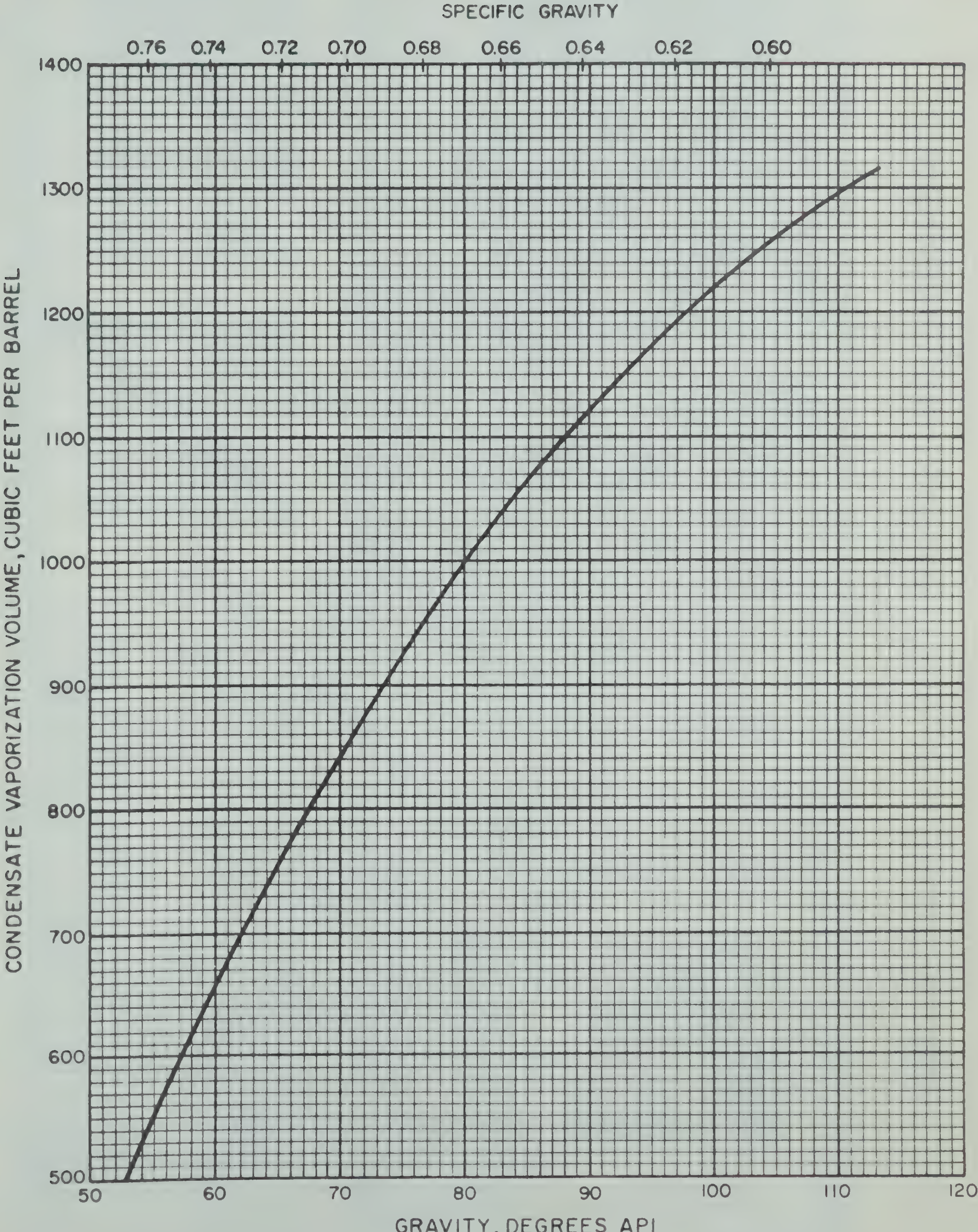


FIGURE A-9 — RELATIONSHIP BETWEEN
CONDENSATE VAPORIZATION VOLUME RATIO AND API GRAVITY.

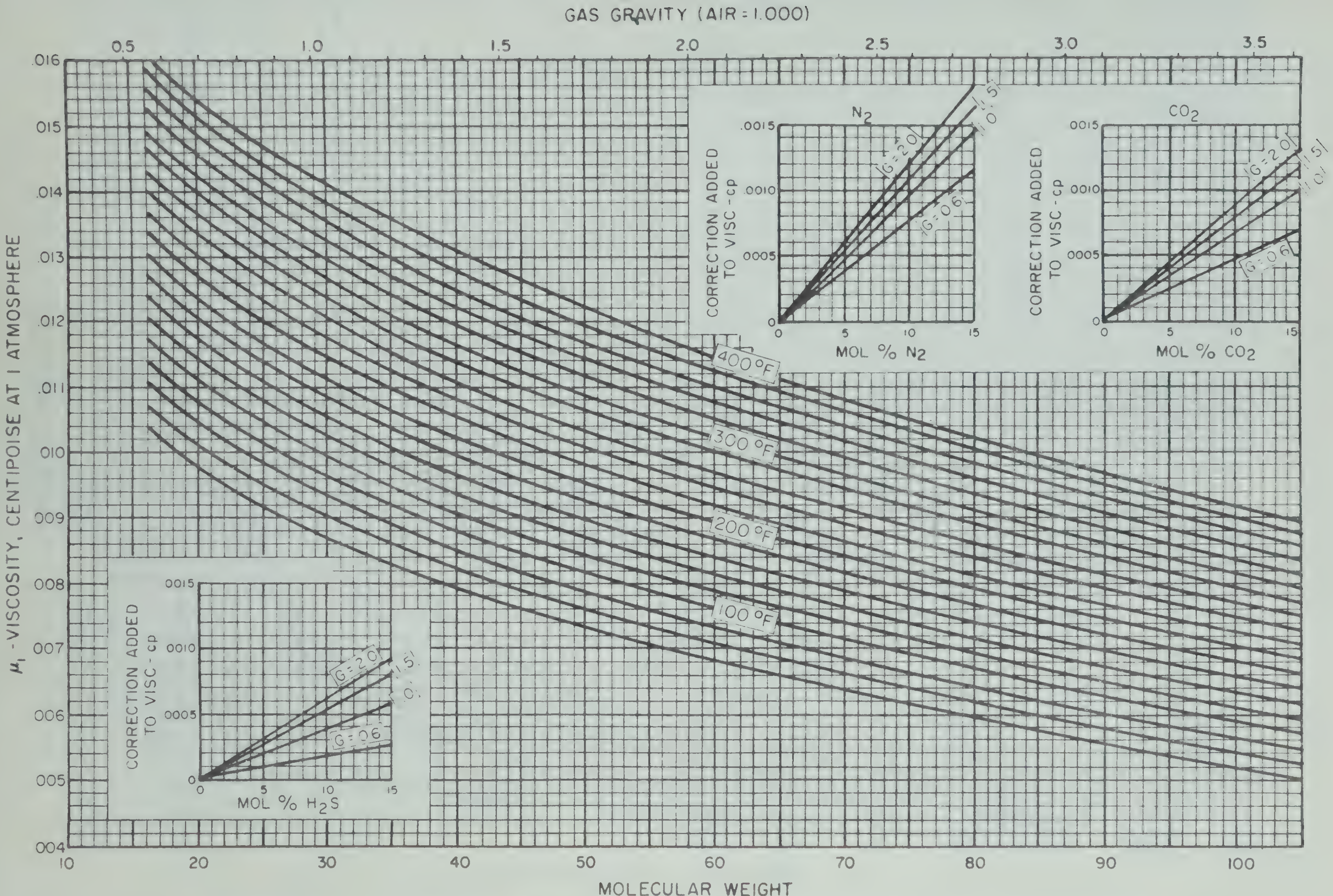


FIGURE A-10 — VISCOSITY OF PARAFFIN HYDROCARBON GASES AT 1 ATMOSPHERE.

(Reproduced from Carr, Kobayashi and Burrows, 13. Courtesy AIME.)

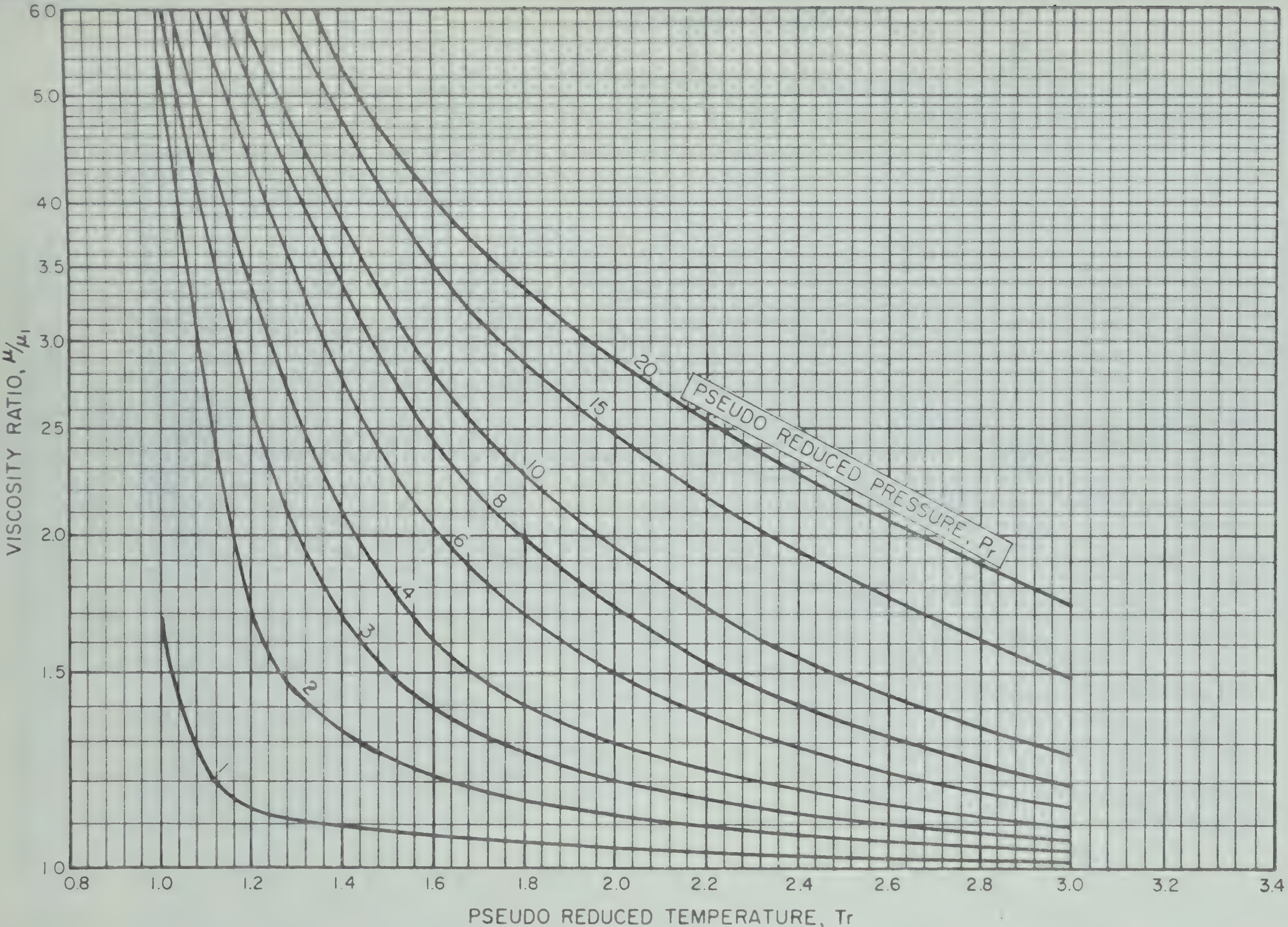


FIGURE A-11 – VISCOSITY RATIO VERSUS PSEUDO REDUCED TEMPERATURE.

(Reproduced from Carr, Kobayashi and Burrows, 13. Courtesy AIME.)

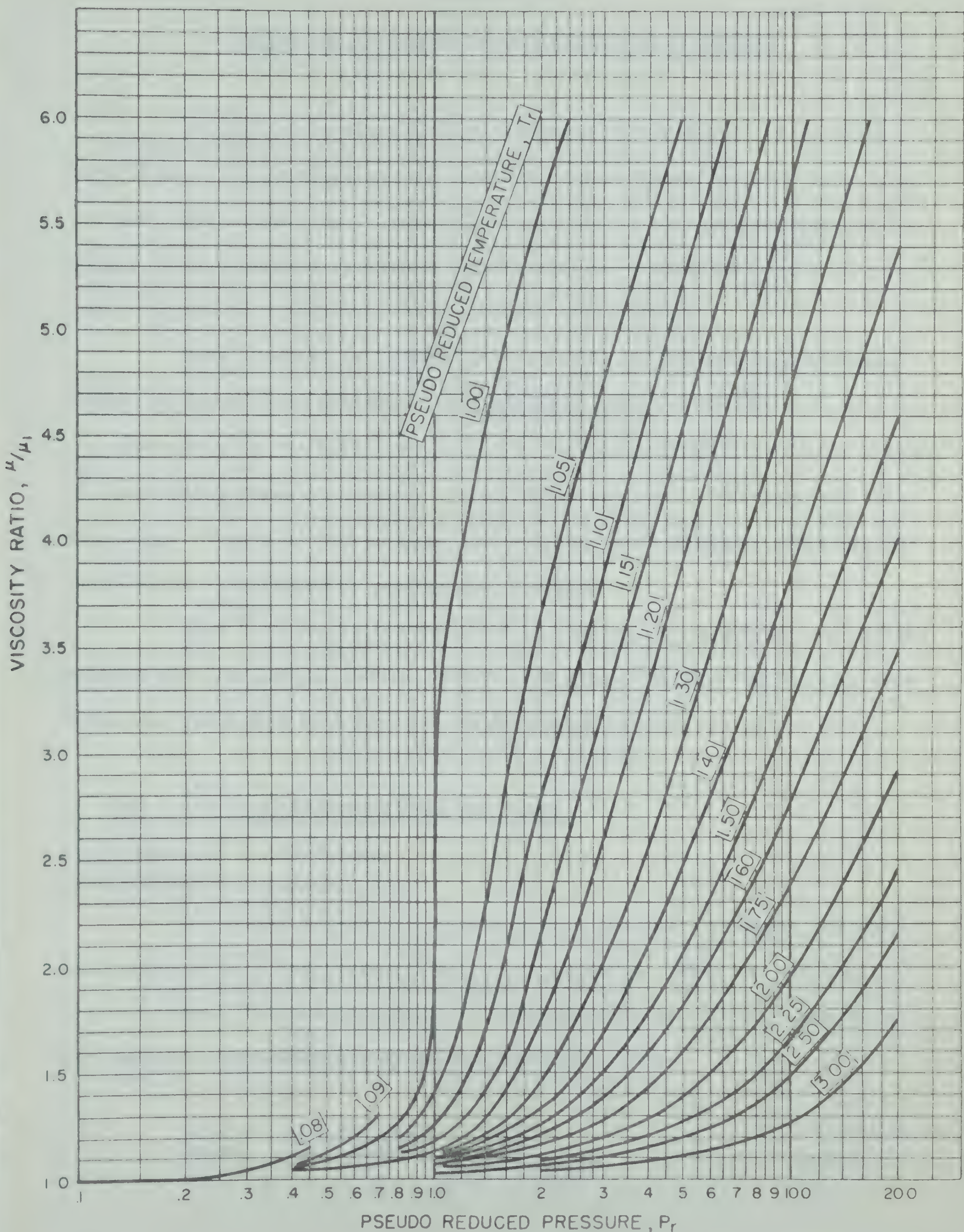


FIGURE A-12 — VISCOSITY RATIO VERSUS PSEUDO REDUCED PRESSURE.

(Reproduced from Carr, Kobayashi and Burrows, 13. Courtesy AIME.)

APPENDIX B

CALCULATION OF BOTTOM HOLE PRESSURES IN GAS WELLS

In order to evaluate the performance of a gas well it is necessary to know the pressure at the sandface of the formation under both flowing and static conditions. Recognizing that it is not always practical to obtain these pressures by direct measurement, it becomes necessary to calculate bottom hole pressures from wellhead observations.

This appendix introduces the basic theory supporting the calculation of bottom hole pressures from wellhead pressures in the case of a gas phase only in the well bore, and also discusses briefly different methods of calculating these pressures. It discusses the main advantages and disadvantages of each of the methods, and sets out a recommended procedure of calculation illustrated by appropriate examples.

Also included in the appendix is a brief discussion of some of the methods for calculating flowing bottom hole pressures from top hole measurements for wells with both gas and liquid phases in the well bore.

A. ONE PHASE (GAS) PRESENT IN THE WELL BORE

Calculation of Static Bottom Hole Pressures

The static bottom hole pressure in a gas well is simply the wellhead pressure plus the pressure due to the weight of the column of gas in the well bore. The starting point for the calculation of this pressure is the familiar mechanical energy balance equation.

$$\frac{144\ dP}{\rho} + dL + \frac{V\ dV}{g_c} + dF = 0 \qquad \text{(B-1)}$$

where

L = height above an arbitrary datum elevation, ft.

ρ = density of gas, lbs - mass per cubic feet.

V = velocity of flow, feet per second.

g_c = dimension conversion factor

$$= 32.17 \frac{(\text{lb-mass}\quad) \text{ ft.}}{(\text{lb. force}) \text{ sec.}^2}$$

dF = energy losses due to irreversibilities (friction).

P = pressure , psia.

The terms in equation (B-1) represent respectively the pressure-volume energy, the potential energy of position, the kinetic energy and the energy losses due to irreversibilities.

For static conditions the kinetic energy and energy losses due to irreversibilities are zero and equation (B-1) may be written as

$$dP = - \frac{\rho}{144} dL \tag{B-2}$$

The density of the gas may be expressed in terms of temperature, pressure, specific gravity and compressibility factor through the Gas Law, as expressed in equation (A-3), and when substituted into equation (B-2) yields

$$\frac{dP}{P} = - \frac{G \, dL}{53.34 \, TZ} \tag{B-3}$$

where

G = specific gravity of the gas.

T = temperature, ^{o}R.

Z = compressibility factor of the gas.

The only assumption involved in this equation is a single phase gas described by the Gas Law. Equation (B-3) is the basis of all methods for calculating static bottom hole pressures from top hole values. The equation is in differential form and must be integrated. The temperature and compressibility of the gas in a well bore varies with the depth, which means that to integrate equation (B-3) a knowledge of the nature of these variations or some assumption regarding them must be made. In addition to this, the gravity of the gas may vary slightly with depth. Since knowledge regarding this variation is seldom available, in the remainder of this appendix it will be assumed the gravity is constant. A number of the methods for integrating this equation, along with the necessary assumptions (assuming only a gas phase in the well bore) are discussed in the following.

The Average Temperature and Compressibility Method

This method includes the well-known Monograph 7 method (72), The Railroad Commission of Texas method (106), and the Vitter method (93). The approach is based on the assumption that both the temperature and compressibility factor are constant at some average values, T_a and Z_a, over the depth of the well.

The average temperature, T_a, is usually taken as the arithmetic average of the reservoir and the wellhead temperature. A refinement which has been proposed (49) is the use of a logarithmetic mean temperature instead of the arithmetic mean. If the compressibility factor were truly constant, and if the temperature profile were linear with depth, the logarithmetic mean temperature would be correct and its use would yield an exact answer. On the other hand, since the compressibility factor does

vary and depends on temperature, there is no assurance that the use of a log mean temperature rather than an arithmetic mean would give a better answer.

There are several ways of calculating an average compressibility factor, but the most common is to use the compressibility factor at the arithmetic average pressure and temperature. This is not necessarily better or worse than using the arithmetic average of the compressibility factors at bottom hole and at top hole conditions.

The average temperature and compressibility method involves a trial and error solution, and may be applied in the form of a one-step calculation from the wellhead to the sandface or a multi-step (usually two) modification. This two step modification involves dividing the well into two equal sections and calculating the pressure at the mid-point of the flow string. The pressure at the sandface is then calculated from the mid-point pressure.

Some of the major advantages of this method are that it is well-known and widely used, and that it may be employed with a sour gas by incorporating the compressibility correction ratios into the calculation. Disadvantages include the necessity of a trial and error calculation and the fact that the method is not accurate where there is a large variation in either the temperature or the compressibility factor. For the latter reason the method is not suitable for deep wells, and is therefore not recommended as a standard procedure for converting top hole pressures to bottom hole conditions for the static case.

The Average Density Method

This method proposed by a University of Alberta graduate class (see Preface), is based on the assumption that the density of the gas in the well bore may be assumed constant at the arithmetic average of the

bottom hole and top hole values. The method is not commonly used, although it has the advantage over the average temperature and compressibility method of introducing only the one overall assumption concerning the key variable (density), as opposed to the two independent assumptions with respect to temperature and compressibility. It may be applied in a one-step or multi-step calculation.

The method involves a trial and error procedure. In cases where the fractional variation of density with depth is appreciably less than the fractional variation of temperature and/or compressibility factor, this method is probably superior to the previously discussed method. However, since the density depends upon the temperature and compressibility factor, the method is not accurate where there is a large variation in either temperature or compressibility; therefore, it is not suitable for deep wells or for adoption as a standard.

The Poettmann Integral Method (68)

This method is based on the assumption that the temperature in the well bore may be assumed constant at some appropriate value with full recognition being given to the variation of the compressibility factor with pressure. Either the arithmetic or the logarithmetic average temperature may be used. For reasons previously discussed there appears to be no assurance that the latter would give a better answer than the former.

The necessary integration of the basic equation has been carried out on a computer. The results are presented in tabular form (49)(68) and may be used directly to solve for a sandface pressure. However, the integration employs compressibility factors which have not been corrected for acid gas constituents, and as a result the method may be used only for sweet gases. Other disadvantages are that the solution requires double

interpolation of integrals within the Poettmann table and that inaccuracies are introduced where there is a large variation in the temperature. The method is not a suitable standard to be applied to all calculations of static bottom hole pressures.

The Cullender and Smith Method (24)

This method enables account to be taken of the variation of both temperature and the compressibility factor with depth. It involves numerical integration for any number of increments down the flow string. The method of solution is a trial and error procedure, and the calculation may be made for a sour gas through incorporation of the appropriate correction ratios. Since each of the previously discussed methods must assume that some variable or combination of variables such as temperature, compressibility factor or density are a constant, it is concluded that this method is theoretically the best available for calculating static bottom hole pressures.

It is recognized that for many wells (particularly those less than four or five thousand feet deep), any of the methods mentioned should result in static bottom hole pressures which are satisfactory. However, since the Cullender and Smith approach is the most suitable for deep wells, it is therefore adopted as the standard method and is discussed in greater detail in this manual.

Cullender and Smith have rearranged equation (B-3) to

$$\frac{GL}{53.34} = \int_{P_w}^{P_s} \frac{TZ\ dP}{P} \qquad \text{(B-4)}$$

where

L = length of flow string, ft.

P_s = the sandface pressure, psia.

P_w = the wellhead pressure, psia.

The right-hand side of the equation may be integrated numerically by determining the average value of (TZ/P) for each of any number of increments in P between P_w and P_s. In general, for limits in pressure of P_o and P_n and designating

$$I = \frac{TZ}{P} \tag{B-5}$$

the right-hand side of equation (B-4) may be expressed as

$$\int_{P_o}^{P_n} \frac{TZ}{P}\, dP = 1/2\, [(P_1 - P_o)(I_1 + I_o) + (P_2 - P_1)(I_2 + I_1) \ldots + (P_n - P_{n-1})(I_n + I_{n-1})]$$

In the case of a two step calculation where only one intermediate value of pressure, that at the mid-depth, P_a is considered, equation (B-4) may be expressed as

$$\frac{GL}{53.34} = \int_{P_w}^{P_s} \frac{TZ}{P}\, dP = \frac{(P_a - P_w)}{2}(I_a + I_w) + \frac{(P_s - P_a)}{2}(I_s + I_a) \tag{B-6}$$

or

$$0.0375\ GL = (P_a - P_w)(I_a + I_w) + (P_s - P_a)(I_s + I_a) \tag{B-7}$$

Equation (B-7) may be separated into two expressions, one for the upper half of the flow string and the other for the lower half. For the upper half

$$0.0375\ G\ \frac{L}{2} = (P_a - P_w)(I_a + I_w) \tag{B-8}$$

and for the lower half

$$0.0375\ G\ \frac{L}{2} = (P_s - P_a)(I_s + I_a) \qquad \text{(B-9)}$$

This relationship is equivalent to assuming the specific volume of gas constant at its arithmetic average value and is the specific volume counterpart to the average density method applied in two steps. While the method may be used with any number of steps, Cullender and Smith have demonstrated that the equivalent of four step accuracy may be obtained with a two step calculation and parabolic interpolation (or the use of Simpson's rule). This results in

$$0.0375\ GL = \frac{(P_s - P_w)}{3}(I_w + 4I_a + I_s) \qquad \text{(B-10)}$$

The solution of equation (B-7) or (B-10) requires knowledge of the temperature at points in the well bore where the quantity, I, is to be evaluated. Except in the rare case where the temperature profile is known, it is necessary to either assume or to calculate one. Lesem et al (53) have presented a method of calculating the temperature-depth relationship, but, sufficient accuracy will normally be attained by assuming the relationship to be linear.

The following procedure is recommended for the solution of equation (B-10).

1. Calculate the left-hand side of equation (B-8) for the upper half of the flow string.

2. Calculate I_w for wellhead conditions from equation (B-5).

3. Assume $I_a = I_w$ for the conditions at the average well depth or at the mid-point of the flow string.

4. Calculate P_a from equation (B-8).

5. Using the value of P_a calculated in step 4 and the

arithmetic average temperature, determine the value of I_a from equation (B-5).

6. Recalculate P_a from equation (B-8) and if this recalculated value does not agree within one psi with the P_a calculated in step 4, repeat the procedure with the new calculated value of P_a until agreement is reached with the previously calculated value.

7. Repeat the entire procedure, using equation (B-9), for the lower half of the well and obtain a value of the sandface pressure.

8. Apply Simpson's rule as expressed in equation (B-10), to obtain a more accurate value of the sandface pressure.

The following example illustrates the use of the Cullender and Smith method.

Example B-1 (A Deep Sweet Gas Well)

A well produces 0.75 gravity gas of a known composition from an average formation depth of 10,000 feet. The shut-in wellhead pressure is 2500 psia. Wellhead and formation temperatures are 35°F and 245°F respectively. Calculate the static bottom hole pressure by the Cullender and Smith method, using a two step calculation and a parabolic interpolation with Simpson's rule.

Solution

$$\left.\begin{array}{ll} P_c & = 667 \text{ psia} \\ T_c & = 408^{\circ}R \end{array}\right\} \text{(From analysis)}$$

$$T_a = \frac{T_w + T_s}{2} = \frac{495 + 705}{2} = 600^{\circ}R$$

$$T_{r_s} = \frac{705}{408} = 1.728$$

$$T_{r_a} = \frac{600}{408} = 1.471$$

$$T_{r_w} = \frac{495}{408} = 1.213$$

$$P_{r_w} = \frac{2500}{667} = 3.748$$

$$Z_w = 0.593$$ (Figure A-2)

Substituting in equation (B-5)

$$I_w = \frac{(495)(0.593)}{(2500)} = 0.1174$$

Solving the left-hand side of equations (B-8) and (B-9)

$$0.0375\ G\ \frac{L}{2} = 0.0375\ (0.75)\ \frac{(10,000)}{2} = 140.63$$

<u>Step 1 (The upper half of the flow string)</u>

<u>First trial</u>

Assume $I_a = I_w = 0.1174$

Solving equation (B-8) for P_a

$$140.63 = (P_a - 2500)(0.1174 + 0.1174)$$

$$P_a = 3099 \text{ psia}$$

<u>Second trial</u>

$$P_{r_a} = \frac{3099}{667} = 4.646$$

$$Z_a = 0.780$$ (Figure A-2)

$$I_a = \frac{(600)(0.780)}{(3099)} = 0.1510$$

Once again solving equation (B-8) for P_a

$$140.63 = (P_a - 2500)(0.1510 + 0.1174)$$

$$P_a = 3024 \text{ psia}$$

Third trial

$$P_{r_a} = \frac{3024}{667} = 4.534$$

$$Z_a = 0.775 \qquad \text{(Figure A-2)}$$

$$I_a = \frac{(600)(0.775)}{(3024)} = 0.1538$$

From equation (B-8)

$$140.63 = (P_a - 2500)(0.1538 + 0.1174)$$

$$P_a = 3019 \text{ psia}$$

Fourth trial

$$P_{r_a} = \frac{3019}{667} = 4.526$$

$$Z_a = 0.775 \qquad \text{(Figure A-2)}$$

$$I_a = \frac{(600)(0.775)}{(3019)} = 0.1540$$

From equation (B-8)

$$140.63 = (P_a - 2500)(0.1540 + 0.1174)$$

$$P_a = 3018 \text{ psia}$$

Step 2 (The lower half of the flow string)

First trial

Assume $I_s = I_a = 0.1540$

From equation (B-9)

$$140.63 = (P_s - 3018)(0.1540 + 0.1540)$$

$$P_s = 3475 \text{ psia}$$

Second trial

$$P_{r_s} = \frac{3475}{667} = 5.210$$

$$Z_s = 0.894$$

$$I_s = \frac{(705)(0.894)}{(3475)} = 0.1814$$

From equation (B-9)

$$140.63 = (P_s - 3018)(0.1540 + 0.1814)$$

$$P_s = 3437 \text{ psia}$$

Third trial

$$P_{r_s} = \frac{3437}{667} = 5.153$$

$$Z_s = 0.892 \qquad \text{(Figure A-2)}$$

$$I_s = \frac{(705)(0.892)}{(3437)} = 0.1830$$

From equation (B-9)

$$140.63 = (P_s - 3018)(0.1540 + 0.1830)$$

$$P_s = 3435 \text{ psia}$$

Simpson's rule is now applied through use of equation (B-10) to improve the accuracy of the calculated sandface pressure.

$$281.26 = \frac{P_s - P_w}{3}(0.1174 + 4(0.1540) + 0.1830)$$

$$P_s - P_w = 921 \text{ psia}$$

$$P_s = 2500 + 921 \text{ psia} = 3421 \text{ psia}$$

For comparative purposes, the same method has been used on the same well with only a one step calculation. This approach involves a direct assumption of bottom hole conditions rather than those at a mid-

point, and trial calculations until the required agreement is reached. The sandface pressure calculated this way is 3436 psia. This and other similar examples have indicated that a one-step calculation would not yield satisfactory results for deep, relatively high pressure wells. On the other hand, a series of calculations for shallow sweet gas wells (less than 5000 feet deep), has shown that a one step calculation can be used without significant loss of accuracy.

The method may also be used to calculate the static bottom hole pressure for a well which produces sour gas. In this case, a correction is applied to the compressibility factors prior to their use in equation (B-5).

To support the theoretically based conclusion that the Cullender and Smith method is the best available, the Board has carried out a series of calculations of bottom hole pressures for a shallow sweet gas well, an intermediate depth sweet gas well, a deep sweet gas well and a deep sour gas well. Although the results of these calculations are not completely conclusive, they support the use of the Cullender and Smith method as a standard for converting static top hole pressures to bottom hole conditions.

Calculation of Flowing Bottom Hole Pressures

The starting point for flowing bottom hole pressure calculations is the previously mentioned mechanical energy balance equation.

$$144 \frac{dP}{\rho} + dL + \frac{VdV}{g_c} + dF = 0 \qquad \text{(B-1)}$$

The kinetic energy term, VdV/g_c, has been shown to be negligible compared to the other terms in the equation and is disregarded in further derivations. The energy loss term, dF, represents the mechanical energy degraded to thermal energy because of irreversibilities attending the flow and is commonly called the friction loss term. In fact, it is the term required to "balance" equation (B-1) and is defined by the equation. It

is convenient to express the friction loss in terms of another quantity, the friction factor*, f, by the well-known Fanning equation

$$dF = \frac{2fV^2dL}{g_c D} \qquad \text{(B-11)}$$

where

f = friction factor.

g_c = dimension conversion factor

$= 32.17 \frac{\text{(lb-mass) ft.}}{\text{(lb. force) sec.}^2}$

D = inside diameter of pipe, ft.

V = velocity of flow, ft. per sec.

The actual lineal velocity of the gas at any point in the well bore, in feet per second, may be related to the production, Q in MMcfd at 14.65 psia and 60°F through the equation

$$Q = \frac{\pi D^2(3600)(24)(520)\ P\ V}{(4)\ 10^6\ (14.65)\ TZ} \qquad \text{(B-12)}$$

which reduces to

$$V = \frac{0.4152\ TZQ}{PD^2} \qquad \text{(B-13)}$$

* A similar friction factor known as the Blasius friction factor is defined by the equation

$$dF = \frac{f_B V^2 dL}{2g_c D}$$

This is commonly used in civil engineering references. It is to be noted that $f_B = 4f$. The Fanning factor is used throughout this manual.

Combining equations (B-11) and (B-13) and the density relationship given in equation (A-3) with equation (B-1) the following is obtained.

$$\frac{53.34\ TZ}{G}\left(\frac{dP}{P}\right) + dL + \frac{2f\,dL}{g_c D}\left(\frac{TZ}{P}\right)^2\left(\frac{0.4152Q}{D^2}\right)^2 = 0 \qquad \text{(B-14)}$$

which reduces to

$$\frac{53.34\ TZ}{G}\left(\frac{dP}{P}\right) + dL + \frac{0.3448\ f\,dL}{g_c D^5}\left(\frac{TZ}{P}\right)^2 Q^2 = 0 \qquad \text{(B-15)}$$

Equation (B-15) may be considered as the basic differential equation relating pressure to flow rate, pipe diameter, vertical position, gas gravity, density and compressibility. The only assumptions in this equation are:

1. Steady state flow.
2. Single phase gas described by the Gas Law.
3. Negligible kinetic energy effects.
4. No work done on or by the gas over the section under consideration.

If 'T' and 'Z' are taken as constant at average values, and in the normal case where 'G' is constant and where there is no choice but to assume 'f' to be constant, the equation may readily be integrated to give

$$Q = 0.1000\sqrt{\frac{(P_s^{\,2} - e^S P_w^{\,2})\ d^5 S}{G T_a Z_a f\ L\ (e^S - 1)}} \qquad \text{(B-16)}$$

where

$$S = \frac{2\ GL}{53.34\ T_a Z_a}$$

L = length of the gas column, feet.

d = inside pipe diameter, inches.

As will be discussed later the assumption of constant 'T' and 'Z' is not a necessary one, but it is inherent in several of the methods to be discussed. For this reason equation (B-16) is sometimes considered the "basic" equation. As compared with equation (B-15) it incorporates the following further assumptions.

5. The gas temperature is constant at some average value, usually taken as the arithmetic average of the terminal values.

6. The compressibility of the gas is constant at some average value, usually taken as that corresponding with the arithmetic average of the terminal pressures and temperatures.

7. The gas gravity is constant (not usually a serious assumption).

8. The friction factor 'f' is constant.

<u>The Friction Factor</u>

Regardless of whether equation (B-15) or equation (B-16) is employed, one of the critical quantities is the friction factor 'f'. This factor is defined by equations (B-1) and (B-11) and is the factor which validates these equations.

Much experimental work has been carried out, particularly in horizontal pipes to determine the variables which influence 'f' and in an effort to develop methods of predicting it. In recent years this work has been augmented by theoretical studies. It is now well established that in the region of turbulent flow, as encountered in gas wells, the friction factor depends upon two quantities, the relative roughness and the Reynolds number.

The relative roughness, δ/d, is the ratio of the absolute roughness, δ (the distance from the peaks to the valleys in pipe wall

irregularities), to the internal pipe diameter 'd'.

The Reynolds number is a grouping of those variables that determine the flow pattern in geometrically similar systems, and may be expressed as

$$N_{Re} = \frac{D\ V\ \rho}{\mu'} \tag{B-17}$$

where

μ' = viscosity of the gas, lb-mass per ft. sec.

For steady state flow this may be expressed in terms of 'Q' to give

$$N_{Re} = \frac{20011\ GQ}{\mu\, d} \tag{B-18}$$

where

G = gas gravity.

Q = flow rate, millions of cubic feet per day at 14.65 psia and 60°F.

μ = viscosity of the gas, centipoise.

d = pipe diameter, inches.

Weymouth (94) developed one of the earliest correlations for the friction factor 'f'. He found that for high rates of flow (high Reynolds numbers) the friction factor depended largely on the pipe diameter and was inversely proportional to the 1/3 power of the diameter.

Weymouth's equation for the friction factor is

$$f = \frac{0.008}{d^{1/3}}$$

This equation does not take account of variations in absolute roughness or of the effect of Reynolds number. At the high Reynolds numbers normally encountered in gas wells, the equations give a friction factor corresponding

to very dirty pipe (see Figure B-2). The Weymouth friction factor is therefore, not suitable for general application.

Probably the best equations now available for the prediction of the friction factor are those based on work by Von Karman (47) and Nikuradse (64), or ones closely related to them. The Colebrook (16) equation

$$\frac{1}{\sqrt{f}} = 4.0 \log \frac{d}{\delta} + 2.28 - 4.0 \log \left(1 + \frac{4.67 \frac{d}{\delta}}{N_{Re} \sqrt{f}} \right) \quad \text{(B-19)}$$

is such a relationship. For values of $\dfrac{\frac{d}{\delta}}{N_{Re} \sqrt{f}}$ less than 0.01 the Reynolds number is found to have no further influence on the friction factor and the equation reduces to

$$\frac{1}{\sqrt{f}} = 4.0 \log \frac{d}{\delta} + 2.28 \quad \text{(B-20)}$$

These equations are awkward to use as such, but, may be represented graphically as in Figure B-1 taken from Knudsen and Katz (51). This figure may be used with confidence to predict the friction factor in cases where the Reynolds number and the relative roughness are known. The Reynolds number presents little problem being readily calculated from equation (B-18), however, the relative roughness is not so easily determined.

Relative Roughness

For clean new pipe the relative roughness is determined by the method of manufacture and usually reflects on absolute roughness, δ, of 0.00055 to 0.0019 (22) (78) (79). For new pipe or tubing used in gas wells the absolute roughness has been found to be in the order of 0.0006 or

0.00065.

A convenient way to present the relative roughness is in the form of a plot of the absolute roughness versus the pipe diameter. Figure B-2 is such a plot and includes curves for well tubing, commercial steel and very dirty pipe. Also included are actual data points reported by Smith and co-workers (78) (79).

Figure B-2 also shows the relative roughness which corresponds to the use of the Weymouth friction factor at four levels of Reynolds numbers, 10^4, 10^5, 10^6 and 10^7. It is to be noted that the Weymouth friction factor corresponds to an unreasonably high relative roughness in small pipes and a high or low relative roughness in intermediate diameter pipes, depending upon whether the Reynolds number is high or low.

The Figure is recommended as a guide in the estimation of the relative roughness. In the absence of special knowledge of the condition of the pipe, the line corresponding with an absolute roughness of 0.00060 inches is recommended.

The Average Temperature and Compressibility Method

This method is similar to the Monograph 7 method, the Railroad Commission of Texas method or the Modified Vitter method, the only real difference being in the evaluation of the friction loss. The method is capable of incorporating any friction factor relationship. However, as employed in the Monograph 7 and Texas Railroad Commission methods, it incorporates the unrealistic Weymouth relationship. The Modified Vitter method also uses the Weymouth factor, but, introduces a compensating correction factor.

Each of the methods incorporate the basic assumption of constant

temperature and compressibility, and as a result is not accurate for deep wells where a large variation in the temperature and compressibility factor may be expected. In addition, where the friction factor has been assumed a function of the pipe diameter only (through use of the Weymouth equation), this will usually introduce additional error into the calculation. If the friction factor used is an appropriate one (such as the Miller (60) corrected Weymouth factor used in the Modified Vitter method, or preferably a friction factor calculated from the Colebrook equation), this method of calculation gives reasonably accurate results for relatively shallow wells. However, since it is not an acceptable approach for deep wells, it cannot be used as a standard method for calculating flowing bottom hole pressures.

The Average Density Method

As in the static pressure case, the flowing calculation by this method is based on the assumption that the density of the gas in the well bore may be assumed constant at the arithmetic average of the bottom hole and top hole values. The friction factor is based on the appropriate relative roughness and the Reynolds number, and is determined from Figure B-1.

As in the static case, this method gives accurate results for relatively shallow wells, but, due to the dependence of the density upon the temperature and compressibility, is not acceptable for deep wells or as a standard procedure. (It may be more accurate than the previously discussed method, if the fractional variation of density with depth is appreciably less than the fractional variation of temperature and/or compressibility).

The Sukkar and Cornell Method (83)

This method is similar to the Poettmann Integral method as presented for the static bottom hole pressure calculation, and is based on the assumption that the temperature in the well bore may be assumed constant at some average value.

The basic equation (B-15) has been solved by numerical integration and the results are presented in tabular form (49) (83). The friction factor as used in this method may be determined from Figure B-1, on the basis of an appropriate relative roughness and the Reynolds number. Since the evaluation of the integral has been based on the Standing-Katz correlation, the method is not applicable for sour gases. Also, the method is not accurate where there is a large variation in temperature (deep wells), so is not acceptable as a standard procedure for calculating flowing sandface pressures.

The Cullender and Smith Method

As in the case of a static well, Cullender and Smith have developed a numerical integration method for calculating flowing bottom hole pressures from wellhead measurements. Again, the method enables account to be taken of the variation in temperature and compressibility factor with depth. It permits a choice of friction factor to suit the relative roughness and the Reynolds number, and also may be used for a sour gas by incorporating compressibility correction ratios into the calculation.

Since this method involves fewer assumptions than any of the other methods, and because it is applicable to a wider range of well conditions, it is adopted as the standard method for calculating bottom

hole pressures and is considered in some detail in the following.

By substituting for the density terms and the friction factor term in the basic differential equation (B-15), Cullender and Smith have developed the following general formula for the calculation of the flowing sandface pressure.

$$\frac{1000\ GL}{53.34} = \int_{P_W}^{P_H} \frac{\frac{P}{TZ}\,dP}{F^2 + \frac{\left(\frac{P}{TZ}\right)^2}{1000}} \qquad \text{(B-21)}$$

where

$$F^2 = \frac{2.6665\ fQ^2}{d^5} \qquad \text{(B-22)}$$

and all other terms are as previously defined.

Equation (B-21) contains no assumptions or approximations other than those of the basic differential equation. As in the case of equation (B-4), the right-hand side of equation (B-21) may be integrated numerically for any number of increments down the flow string. Precise solution requires a detailed knowledge of the temperature profile. Except in the rare case where it is known, the profile is normally assumed to be linear.

Equation (B-22) may be solved for 'F' directly by determining the average viscosity (at the average temperature and pressure), the Reynolds number and the friction factor. However, a simplification based on an absolute roughness of 0.0006 inches, as calculated from the Nikuradse friction factor equation for fully developed turbulent flow gives

$$F_r Q = F = \frac{0.10797\ Q}{d^{2.612}} \qquad \text{(B-23)}$$

for internal pipe diameters less than 4.277 inches and

$$F_r Q = F = \frac{0.10337\ Q}{d^{2.582}} \qquad \text{(B-24)}$$

for internal pipe diameters greater than 4.277 inches.

Values of F/Q (called F_r by Cullender and Smith and calculated from equations (B-23) and (B-24)) for various tubing and casing sizes are presented in Table B-1.

Equation (B-21) may be solved approximately by numerical integration between the limits P_w and P_s by employing a two step procedure using an average well pressure, P_a, at half the well depth.

The equation may then be written as

$$\frac{1000\ GL}{53.34} = \int_{P_w}^{P_s} \frac{\frac{P}{TZ}\,dP}{F^2 + \frac{\left(\frac{P}{TZ}\right)^2}{1000}} = \frac{(P_a - P_w)}{2}(I_a + I_w) + \frac{(P_s - P_a)}{2}(I_s + I_a) \qquad \text{(B-25)}$$

$$\text{or } 37.5GL = (P_a - P_w)(I_a + I_w) + (P_s - P_a)(I_s + I_a) \qquad \text{(B-26)}$$

where

$$I = \frac{\frac{P}{TZ}}{F^2 + \frac{\left(\frac{P}{TZ}\right)^2}{1000}} \qquad \text{(B-27)}$$

Equation (B-26) may be compared to equation (B-7) which was developed for static bottom hole pressure calculations.

A two step procedure and the use of Simpson's rule will also give the equivalent of four step accuracy for the calculation of the flowing bottom hole pressures. For the following case, the use of Simpson's rule

results in

$$37.5GL = \frac{(P_s - P_w)}{3} (I_w + 4I_a + I_s) \qquad \text{(B-28)}$$

Since a two step procedure will normally be used, it is convenient to divide equation (B-26) into two expressions, one for the upper half and the other for the lower half of the flow string. For the upper half

$$37.5 G \frac{L}{2} = (P_a - P_w)(I_a + I_w) \qquad \text{(B-29)}$$

and for the lower half

$$37.5 G \frac{L}{2} = (P_s - P_a)(I_s + I_a) \qquad \text{(B-30)}$$

As in the static calculation the assumption that the temperature gradient is linear in the well bore is also used to solve equation (B-26) or (B-28) for the flowing bottom hole pressure.

The following general procedure is recommended for solving these equations.

1. Calculate $37.5 G \frac{L}{2}$, (for the upper half of the flow string).
2. Calculate F^2 from equation (B-23) or (B-24).
3. Determine I_w for wellhead conditions, from equation (B-27).
4. Assume $I_a = I_w$ for the conditions at the average well depth.
5. Calculate P_a from equation (B-29).
6. Using the value of P_a calculated in step 5 and the arithmetic average temperature, determine the value of I_a from equation (B-27).
7. Recalculate P_a from equation (B-29) and if this recalculated value does not agree within one psi with the P_a calculated in step 5, repeat the procedure with the new calculated value of P_a until agreement is reached with the previously calculated value.
8. Repeat the entire procedure using equation (B-30) for the lower half of the well and obtain a value of the sandface pressure.
9. Apply Simpson's rule as expressed in equation (B-28) to

obtain a sandface pressure equivalent to a four step calculation. (It should once again be noted that for wells less than 5000 feet deep, a one step calculation can be used without significant loss of accuracy.)

The following example illustrates the use of this method.

Example B-2 (A Deep Sweet Gas Well)

A gas well produces 0.75 gravity gas of a known composition from an average formation depth of 10,000 feet through 2.441 I.D. tubing. The wellhead pressure at a stabilized flow rate at 4.915 MMcfd is 2000 psia. The flowing wellhead temperature is 110°F and the formation temperature is 245°F. Calculate the flowing sandface pressure by the Cullender and Smith method using two step calculation and adjustment with Simpson's rule.

Solution

$$\left.\begin{array}{ll} P_c & = 667 \text{ psia} \\ T_c & = 408°R \end{array}\right\} \quad \text{(From analysis)}$$

$$37.5\ G\ \frac{L}{2} = 140{,}625$$

From equation (B-23)

$$F = \frac{(0.10797)\ (4.915)}{(2.441)^{2.612}} = 0.05158$$

$$F^2 = 0.002660$$

$$T_{r_w} = \frac{570}{408} = 1.397$$

$$P_{r_w} = \frac{2000}{667} = 2.999$$

$$Z_w = 0.705 \qquad \text{(Figure A-2)}$$

$$T_a = \frac{570 + 705}{2} = 638°R$$

$$T_{r_a} = \frac{638}{408} = 1.564$$

$$T_{r_s} = \frac{705}{408} = 1.728$$

$$\left(\frac{P}{TZ}\right)_w = \frac{2000}{(570)(0.705)} = 4.977$$

From equation (B-27)

$$I_w = \frac{4.977}{(0.002660) + \frac{(4.977)^2}{1000}} = 181.4$$

Step 1 (The upper half of the flow string)

First trial

Assume $I_a = I_w$

Solving equation (B-29) for P_a

$$140,625 = (P_a - 2000)(181.4 + 181.4)$$

$$P_a = 2388 \text{ psia}$$

Second trial

$$P_{r_a} = \frac{2388}{667} = 3.580$$

$$Z_a = 0.800 \qquad \text{(Figure A-2)}$$

$$\left(\frac{P}{TZ}\right)_a = \frac{2388}{(638)(0.800)} = 4.679$$

From equation (B-27)

$$I_a = \frac{4.679}{(0.002660) + \frac{(4.679)^2}{1000}} = 190.6$$

Solving equation (B-29)

$$140{,}625 = (P_a - 2000)(190.6 + 181.4)$$

$$= 2378 \text{ psia}$$

Third trial

$$P_{r_a} = \frac{2378}{667} = 3.565$$

$$Z_a = 0.800 \qquad \text{(Figure A-2)}$$

$$\left(\frac{P}{TZ}\right)_a = \frac{2378}{(638)(0.800)} = 4.659$$

From equation (B-27)

$$I_a = \frac{4.659}{(0.002660) + \frac{(4.659)^2}{1000}} = 191.2$$

Solving equation (B-29)

$$140{,}625 = (P_a - 2000)(191.2 + 181.4)$$

$$P_a = 2377 \text{ psia}$$

Step 2 (The lower half of the flow string)

First trial

Assume $I_s = I_a$

Solving equation (B-30) for P_s

$$140{,}625 = (P_s - 2377)(191.2 + 191.2)$$

$$P_s = 2745 \text{ psia}$$

Second trial

$$P_{r_s} = \frac{2745}{667} = 4.115$$

$$Z_s = 0.869 \qquad \text{(Figure A-2)}$$

$$\left(\frac{P}{TZ}\right)_s = \frac{2745}{(705)(0.869)} = 4.481$$

From equation (B-27)

$$I_s = \frac{4.481}{(0.002660) + \frac{(4.481)^2}{1000}} = 197.1$$

Solving equation (B-30)

$$140{,}625 = (P_s - 2377)(197.1 + 191.2)$$

$$P_s = 2739 \text{ psia}$$

Third trial

$$P_{r_s} = \frac{2739}{667} = 4.106$$

$$Z_s = 0.869 \qquad \text{(Figure A-2)}$$

$$\left(\frac{P}{TZ}\right)_s = \frac{2739}{(705)(0.869)} = 4.471$$

From equation (B-27)

$$I_s = \frac{4.471}{(0.002660) + \frac{(4.471)^2}{1000}} = 197.4$$

Solving equation (B-30)

$$140{,}625 = (P_s - 2377)(197.4 + 191.2)$$

$$P_s = 2739 \text{ psia}$$

Simpson's rule is now applied through use of equation (B-28) to improve the accuracy of the calculated pressure.

$$281{,}250 = \frac{P_s - P_w}{3} [181.4 + 4\ (191.2) + 197.4]$$

$$P_s - P_w = 737.8 \text{ psia}$$

$$P_s = 2000 + 737.8 = 2738 \text{ psia}$$

The method may also be used for calculating the flowing sandface pressure for a well which produces sour gas by applying corrections to the compressibility factors prior to their use in equation (B-27).

A series of calculations confirm the theoretical soundness of the Cullender and Smith method. To really determine which method is best for practical application, would require extensive field measurement of both top hole and bottom hole pressures. Such measurements as are presently available to the Board lend support to the Cullender and Smith method, but adequate information of this nature is not available. Since the method is applicable under most conditions and because of its theoretical aptness, it is adopted as a standard for converting top hole pressures to bottom hole conditions.

Annular Flow

In most cases where the well is flowing in the annulus between the casing and the tubing, it is possible to measure the corresponding shut-in tubing pressure. The flowing sandface pressure may then be calculated by the previously discussed method (Cullender and Smith) for determining the static bottom hole pressure.

However, it is on occasion necessary to calculate the flowing sandface pressure of an annular column from the flowing wellhead pressure. Rigorous equations for the determination of the friction loss term for

flow in an annulus are not available. It is necessary to use the equations for flow in a circular pipe, incorporating an effective diameter for the annular space. This is commonly done through the use of the hydraulic radius formula which may be written as

$$D_{eff} = 4 \, \frac{\text{cross-sectional area of flow}}{\text{wetted perimeter}} \tag{B-31}$$

which for the annulus reduces to

$$D_{eff} = D_2 - D_1 \tag{B-32}$$

where

D_2 = inside diameter of casing, ft.

D_1 = outside diameter of tubing, ft.

Substituting the effective diameter in the friction loss term, equation (B-11) may be written as

$$dF = \frac{2fV^2 dL}{g_c \, (D_2 - D_1)} \tag{B-11a}$$

Equation (B-13) for the tubing gas velocity may be written for the annular gas velocity as

$$V = \frac{0.4152 \; TZQ}{P \, (D_2^2 - D_1^2)} \tag{B-13a}$$

It should be noted that the Reynolds number is determined from

$$N_{Re} = \frac{20,011 \; GQ}{(d_2 + d_1)\mu} \tag{B-18a}$$

as compared to equation (B-18).

Substituting equations (B-11a) and (B-13a) into the basic flow equation gives

$$\frac{53.34TZ}{G}\frac{dP}{P} + dL + \frac{2fdL}{g_c(D_2 - D_1)}\left(\frac{TZ}{P}\right)^2\left(\frac{0.4152Q}{D_2^{\,2} - D_1^{\,2}}\right)^2 = 0 \qquad \text{(B-14a)}$$

Equation (B-14a) may be integrated by any of the methods previously discussed, and results in equations which are then used to calculate flowing bottom hole pressures for the case of annular flow.

Equations (B-23) and (B-24) as written for annular flow, and with the diameter in inches, are as follows.

$$F = \frac{0.10797\ Q}{(d_2 - d_1)^{1.612}\ (d_2 + d_1)} \qquad \text{(B-23a)}$$

for effective diameters less than 4.277 inches, and

$$F = \frac{0.10337\ Q}{(d_2 - d_1)^{1.582}\ (d_2 + d_1)} \qquad \text{(B-24a)}$$

for effective diameters greater than 4.277 inches.

B. TWO PHASES (GAS AND LIQUID) PRESENT IN THE WELL BORE

The preceeding section has dealt with the calculation of bottom hole pressures from wellhead measurements, for the case where only one phase (gas) is present in the well bore. With the trend towards the discovery of deep, high pressure, retrograde-type, condensate reservoirs, the case where both a gas and a liquid phase exists in the well bore becomes increasingly important. Also, where wells are producing significant amounts of free water from the formation, consideration should be given to the effect of this liquid phase on the calculation of sandface pressures.

In the static case, if two phases exist in the well bore, the liquid phase will exist as a "liquid leg" at the bottom of the flow string. In this situation the bottom hole pressure must be measured

directly with a bottom hole pressure gauge, or calculated from a knowledge of the level of the liquid in the well bore and the gradient within the liquid phase. In the latter case, the pressure is calculated from the wellhead down to the interface by the normal means for a gas well, and then the bottom hole pressure is calculated from a knowledge of the liquid gradient. Since the level of the liquid and its gradient are seldom known, the normal means of determining the static bottom hole pressure in the two phase case, is by direct measurement.

For the two phase flowing case, in addition to the possibility of direct measurement, there are a number of methods available for calculating the flowing sandface pressure from wellhead measurements. A brief discussion of some of these methods is included in this section.

The first thing that must be decided when testing a well in a gas-condensate type reservoir is whether or not two phases exist in the well bore. This is accomplished by determining the phase condition of the well bore fluid through a calculation of the hydrocarbon dewpoint temperature at the flowing wellhead pressure. The dewpoint is the condition at which the first droplet of liquid forms, and if the dewpoint temperature is found to be above the flowing wellhead temperature this indicates that liquid exists and two phase flow is taking place in the well bore. If two phases are found to exist, equilibrium flash calculations can be made to determine the relative amounts and the composition of each phase at any point in the well bore.

To calculate the wellhead dewpoint temperature and composition of phases in the well bore, use is made of vapor-liquid equilibrium ratios. The vapor-liquid equilibrium ratio for a component in the fluid is defined by the equation

$$K_i = \frac{y_i}{x_i} \tag{B-33}$$

where

y_i = mole fraction of component i in the vapor phase.

x_i = mole fraction of component i in the liquid phase.

The value of K_i for each component depends on pressure, temperature and the composition of the mixture.

Many correlations have been presented for the determination of K_i values (48) (74) (81). All of the recent correlations incorporate some composition variable to account for the distribution of components in the mixture.

The method of calculating dewpoint temperatures and the composition of phases are presented in many references (49). A convenient consolidation of the method, along with illustrative examples have been presented by Van Wielingen (92), for calculating the dewpoint temperature of a natural gas, the relative amounts of the gas and liquid phases that will exist when a mixture of gaseous hydrocarbons is subjected to a specific temperature and pressure, and the composition of each of the phases at these conditions.

A study of production from gas-condensate reservoirs in the Province of Alberta has been made in an attempt to correlate the existence of two-phase flow conditions to the liquid content of the fluid. The study has resulted in the conclusion that if the products of a high pressure separator (operating under average conditions) are such that the liquid - gas ratio is less than about 30 barrels of liquid hydrocarbons per million cubic feet of gas, there is little likelihood that two-phase flow occurs in the well bore. On the other hand, if the ratio is greater

than about 60 barrels per million cubic feet, it is very likely that two-phase flow will exist. It must be stressed that the foregoing are merely generalizations and that many exceptions to the rule may be encountered in practice. For this reason, this rule is not intended to replace calculations to determine the phase condition.

When the vertical flow of gas-liquid mixtures does take place, it differs materially from one phase flow, in two important respects.

1. The flow pattern is much more complex, and depends on the diameter of the flow tube and the properties and relative amounts of the phases.

2. The composition of the total fluid in the flow tube differs from that produced because of the difference in densities of the two phases. The less dense phase tends to slip past the heavier phase and thus the heavier phase accumulates in the flow tube. This phenomenon is generally referred to as "slip" or "holdup", and in the case of a well producing liquid is represented by the gas slipping past the liquid with a resulting higher liquid-gas ratio in the flow tube.

Many authors have described the flow patterns for the upward flow of gas-liquid mixtures using various terminology. Gosline (32) described the patterns as changing from dispersed gas through gas piston to dispersed liquid, as increasing quantities of gas are added to a flowing liquid column. Martinelli et al (56) described the same range of patterns as changing from liquid viscous - gas viscous through liquid viscous - gas turbulent to liquid turbulent - gas turbulent. Govier, Radford and Dunn (33) described the two phase flow patterns based upon visual examination and correlated their description to pressure drop observations. For convenience, the patterns as described by Govier et al are adopted and used in this discussion.

As gas is introduced into a column of liquid flowing at a constant rate, the gas disperses as bubbles which increase in number and size. This is the bubble flow pattern. With an increase in gas flow, the bubbles coalesce and form bullet shaped slugs characteristic of the slug flow pattern. As the gas-liquid ratio is increased further, the slugs merge and appear to degenerate into froth. A further increase in the gas-liquid ratio results in less relative motion between phases, and the liquid usually forms a ripply layer moving upwards on the tube wall. This is termed ripple flow. A further increase in the gas flow rate results in a smoothing of the waves or ripples on the flow wall with gas flowing at very high rates through the inner core. This pattern is termed film flow. Eventually, at an even higher gas flow rate, the entire liquid layer will be captured from the wall, and carried in a dispersed state in the gas. This is known as the mist flow region.

It should be noted that not all of these patterns can be observed under all flow conditions. In particular, where the liquid flow rate is high, the increase in gas-liquid ratio required to move from region to region is considerably less, and the transition from pattern to pattern is not so distinguishable.

The complexity of the flow pattern and the existence of holdup greatly alter the relationship between pressure drop and pertinent variables for two phase flow when compared to single phase flow. To illustrate, pressure losses in two phase flow do not always increase with an increase in the flow rate or with a decrease in the flow tube size. Occasionally, increased gas flow rates without significant increase in liquid flow rates, will serve to reduce the holdup and the resulting portion of the gradient due to the weight of the column without greatly increasing the friction loss. This results in an overall reduction in pressure drop with the increasing flow rate. Continued increases in

the gas flow rate, eventually result in increasing friction losses which more than offset the decreasing weight of the column. The pressure loss-production rate relationship then reverses and is similar to the single phase relationship.

This phenomenon is illustrated in Figure B-3, taken from the work of Govier, Radford and Dunn.

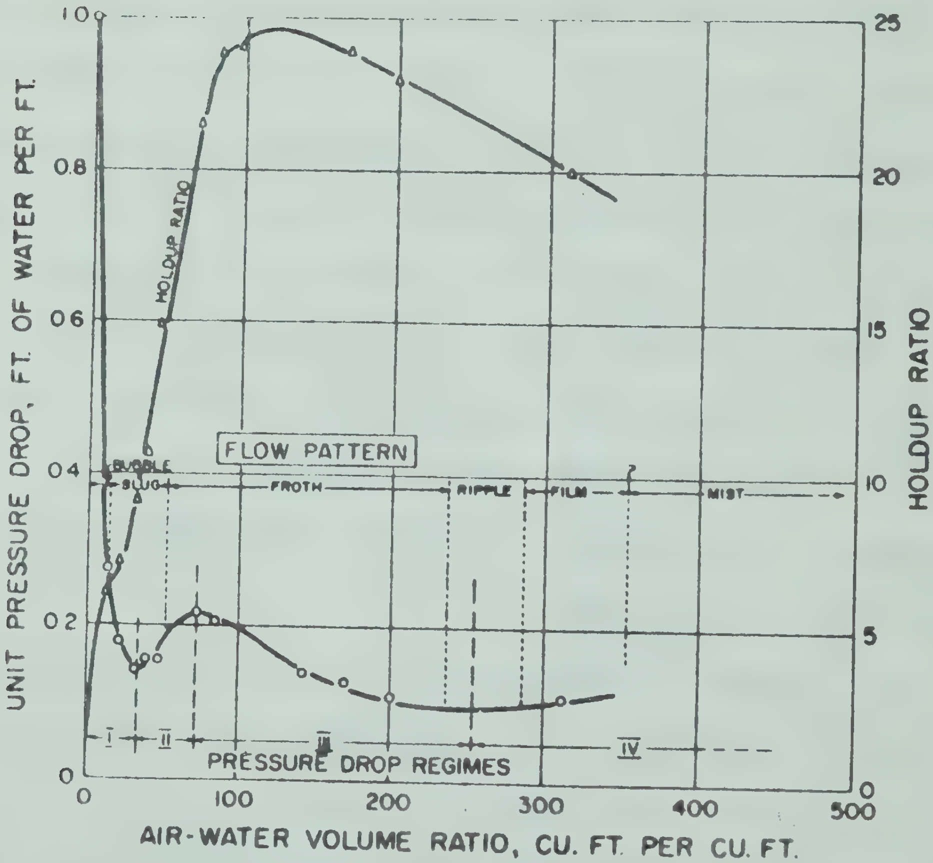


FIGURE B-3 — TYPICAL TWO-PHASE PRESSURE DROP CURVE.
(Redrawn from Govier et al(33)

Many attempts have been made to correlate the flow patterns in a two phase system, and to evaluate the pressure drop in each region (11)(33)(34)(37)(75). All of these investigations have been based on laboratory studies at relatively low pressures. In addition, a number of attempts (6)(38)(69) have been made to determine the two phase pressure drop based on actual field conditions. Unfortunately, these investigations have been concerned primarily with the case of gas-lifting in oil wells (under conditions of bubble and slug flow), and have not been extended into flow regions of particular interest in the producing of gas-condensate wells. Use of the laboratory based correlations indicate that flow patterns for a typical gas-condensate well, in general are in the range of ripple, film or mist, but the different correlations are not in full agreement.

On the basis of a general review of these methods, it has been concluded that the Ros (75) approach is probably the most accurate method now available for application to gas-condensate field systems, both with respect to the correlation of flow patterns and to the determination of the pressure drop. The reader is referred to the original paper for details of the method.

Conclusions

There are several laboratory based methods available for calculating flowing bottom hole pressures when two phases exist in the well bore. These are not consistent amongst themselves, and none have been extensively tested for actual field gas-condensate systems. For this reason, and because of the likelihood that for gas-condensate wells, two phase flow exists for only a portion of the flow string, it is recommended that whenever two phase flow is suspected the flowing bottom hole pressures be directly measured during initial testing, until accurate procedures or correlations are developed for calculating sub-surface pressures.

TABLE B-1

VALUES OF F_r FOR VARIOUS TUBING AND CASING SIZES
(From Cullender and Smith (24))

(Use only for internal diameters less than 4.277 in.)

$$F_r = \frac{0.10797}{d^{2.612}}$$

Nominal Size, In.	O.D. In.	Lb. Per Ft.	I.D. In.	F_r
1	1.315	1.80	1.049	0.095288
1-1/4	1.660	2.40	1.380	0.046552
1-1/2	1.990	2.75	1.610	0.031122
2	2.375	4.70	1.995	0.017777
2-1/2	2.875	6.50	2.441	0.010495
3	3.500	9.30	2.992	0.006167
3-1/2	4.000	11.00	3.476	0.004169
4	4.500	12.70	3.958	0.002970
4-1/2	4.750	16.25.	4.082	0.002740
	4.750	18.00	4.000	0.002889
4-3/4	5.000	18.00	4.276	0.002427
	5.000	21.00	4.154	0.002617

(Use only for internal diameters greater than 4.277 in.)

$$F_r = \frac{0.10337}{d^{2.582}}$$

4-3/4	5.000	13.00	4.494	0.0021345
	5.000	15.00	4.408	0.0022437
5-3/16	5.500	14.00	5.012	0.0016105
	5.500	15.00	4.976	0.0016408
	5.500	17.00	4.892	0.0017145
	5.500	20.00	4.778	0.0018221
	5.500	23.00	4.670	0.0019329
	5.500	25.00	4.580	0.0020325
5-5/8	6.000	15.00	5.524	0.0012528
	6.000	17.00	5.450	0.0012972
	6.000	20.00	5.352	0.0013595
	6.000	23.00	5.240	0.0014358
	6.000	26.00	5.140	0.0015090
6-1/4	6.625	20.00	6.049	0.0009910
	6.625	22.00	5.989	0.0010169
	6.625	24.00	5.921	0.0010473
	6.625	26.00	5.855	0.0010781
	6.625	28.00	5.791	0.0011091

TABLE B-1 (cont.)

Nominal Size, In.	O.D. In.	Lb. Per Ft.	I.D. In.	F_r
6-1/4(cont.)	6.625	31.80	5.675	0.0011686
	6.625	34.00	5.595	0.0012122
6-5/8	7.000	20.00	6.456	0.0008876
	7.000	22.00	6.398	0.0008574
	7.000	24.00	6.336	0.0008792
	7.000	26.00	6.276	0.0009011
	7.000	28.00	6.214	0.0009245
	7.000	30.00	6.154	0.0009479
	7.000	40.00	5.836	0.0010871
7-1/4	7.625	26.40	6.969	0.0006875
	7.625	29.70	6.875	0.0007121
	7.625	33.70	6.765	0.0007424
	7.625	38.70	6.625	0.0007836
	7.625	45.00	6.445	0.0008413
	8.000	26.00	7.386	0.0005917
7-5/8	8.125	28.00	7.485	0.0005717
	8.125	32.00	7.385	0.0005919
	8.125	35.50	7.285	0.0006132
	8.125	39.50	7.185	0.0006354
8-1/4	8.625	17.50	8.249	0.0004448
	8.625	20.00	8.191	0.0004530
	8.625	24.00	8.097	0.0004667
	8.625	28.00	8.003	0.0004810
	8.625	32.00	7.907	0.0004962
	8.625	36.00	7.825	0.0005098
	8.625	38.00	7.775	0.0005183
	8.625	43.00	7.651	0.0005403
8-5/8	9.000	34.00	8.290	0.0004392
	9.000	38.00	8.196	0.0004523
	9.000	40.00	8.150	0.0004589
	9.000	45.00	8.032	0.0004765
9	9.625	36.00	8.921	0.0003634
	9.625	40.00	8.835	0.0003726
	9.625	43.50	8.755	0.0003814
	9.625	47.00	8.681	0.0003899
	9.625	53.50	8.535	0.0004074
	9.625	58.00	8.435	0.0004200
9-5/8	10.000	33.00	9.384	0.0004167
	10.000	55.50	8.908	0.0003648
	10.000	61.20	8.790	0.0003775
10	10.750	32.75	10.192	0.0002576
	10.750	35.75	10.136	0.0002613
	10.750	40.00	10.050	0.0002671
	10.750	45.50	9.950	0.0002741
	10.750	48.00	9.902	0.0002776
	10.750	54.00	9.784	0.0002863

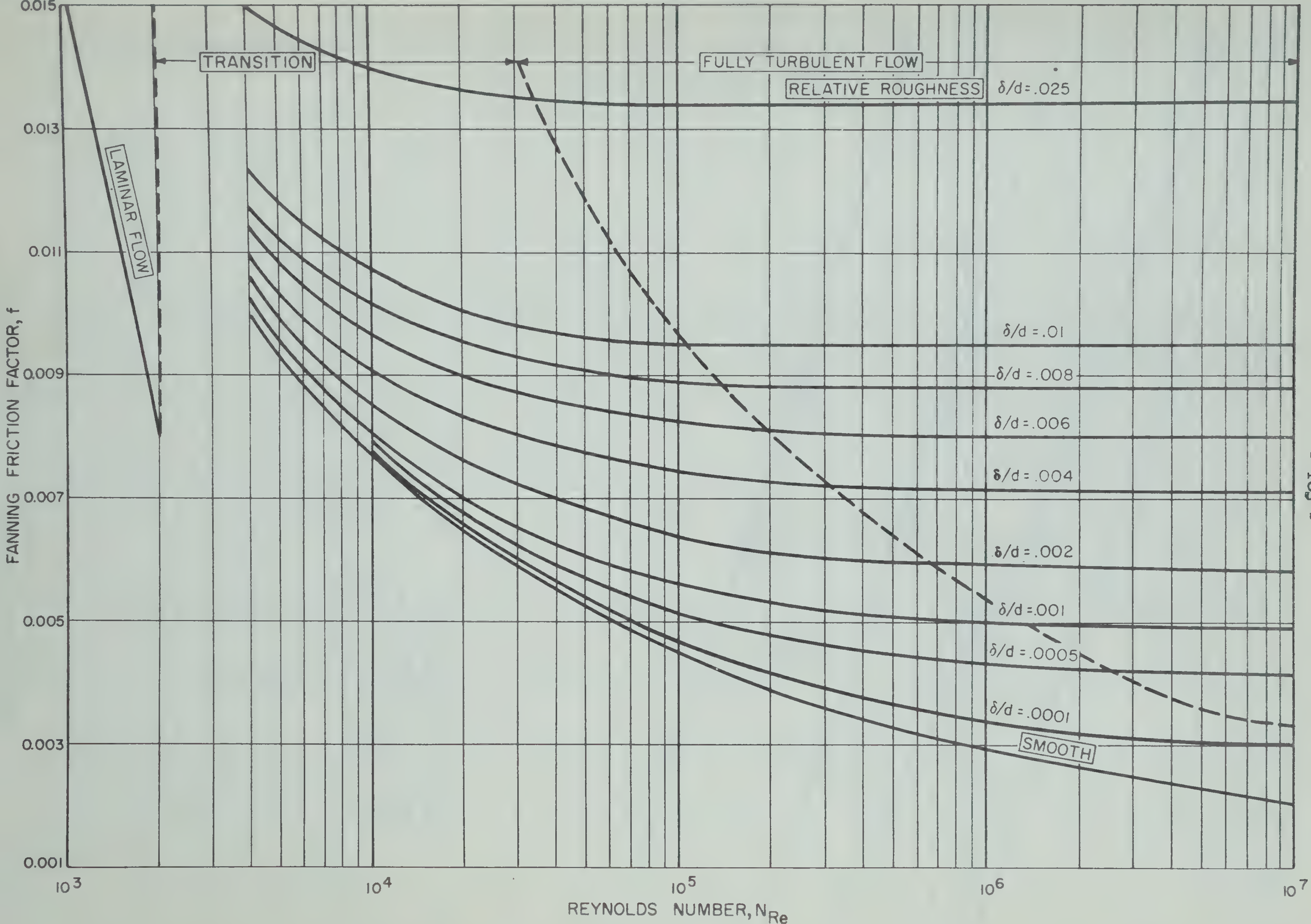


FIGURE B-1 — FRICTION FACTOR FOR FLUID FLOW IN PIPES

(Modified from "Fluid Dynamics and Heat Transfer," by Knudsen and Katz (51) Copyright 1958. McGraw-Hill Book Company; used by permission.)

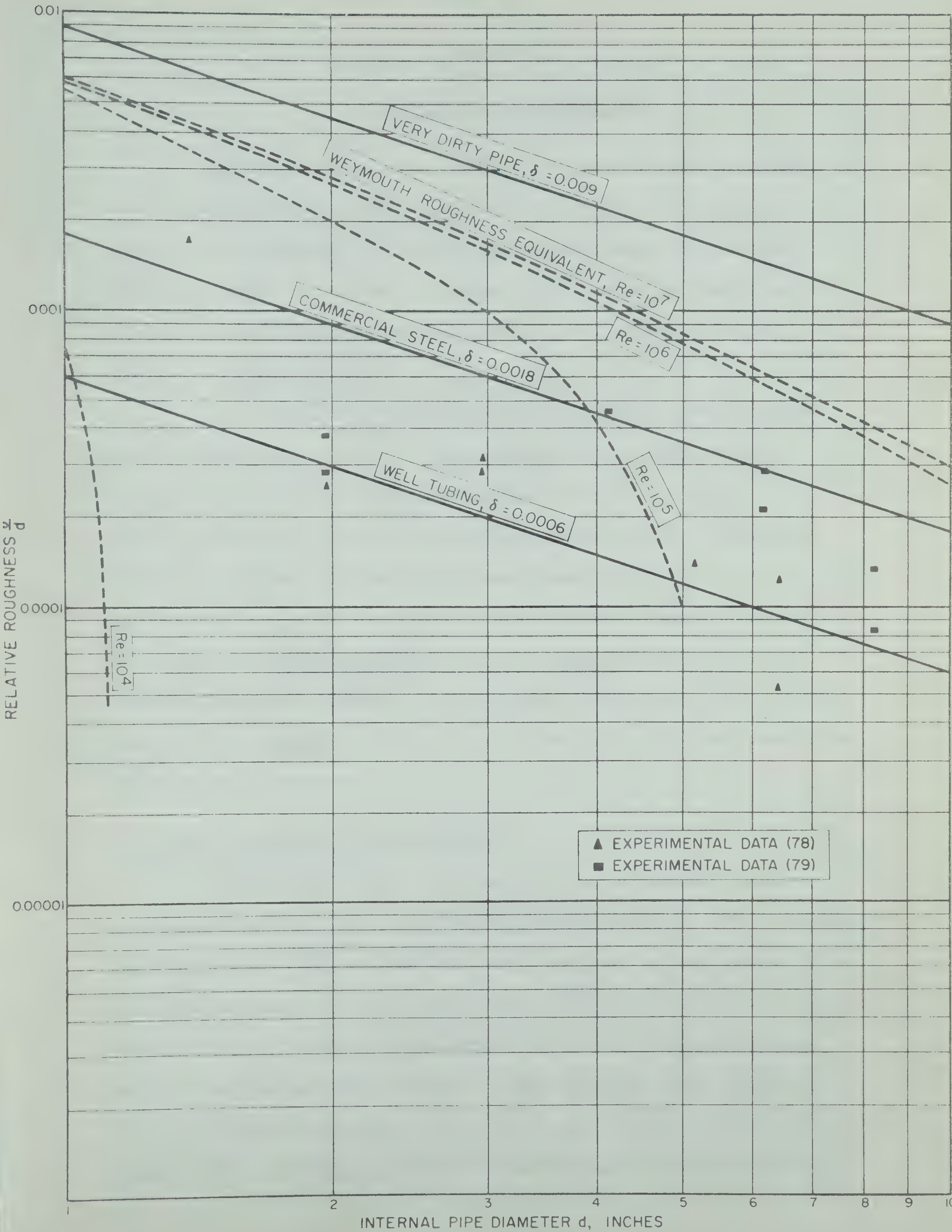


FIGURE B-2 – RELATIVE ROUGHNESS OF PIPES.

APPENDIX C

GAS FLOW IN THE RESERVOIR

Gas flow through porous media in a reservoir may be steady-state or unsteady-state. Steady-state flow requires that conditions (flow rate and pressure) do not change with time while for unsteady-state flow, these conditions do change. For example, unsteady-state producing operations will result in a decline in both the flow rate and pressure, with time.

Katz, et al (49) have shown that even though pressure depletion of a gas pool is normal in an unsteady-state process, flow may be treated as steady-state in regions near the well bore under certain conditions, or after the well has flowed at stable conditions for lengthy periods of time. Under these conditions, the rate of decline in flow rate and pressure will be constant over the reservoir and will be very slow, reflecting the ratio of the withdrawals in a set time period to the total volume of the reservoir. This type of flow is often referred to as pseudo steady-state and is usually treated as steady-state for calculation purposes.

A. STEADY-STATE FLOW IN THE RESERVOIR

Steady-state flow in the reservoir may be laminar, turbulent, or a mixture of both. This was initially demonstrated in 1933 by Fancher, Lewis and Barnes (28), and has since been confirmed by many others (8)(9)(12)(36). Fancher, Lewis and Barnes made extensive measurements on the pressure drop - flow rate relationships for the flow of gases through cores of various permeable materials. They expressed their results in terms of a modified Fanning friction factor and a modified Reynolds number (both incorporating the grain diameter) and defined respectively

$$f_{FLB} = \frac{g_c D_g (\Delta P)}{2 V_a^2 \rho L} \tag{C-1}$$

and

$$N_{ReFLB} = \frac{D_g V_a \rho}{\mu'} \tag{C-2}$$

where

ΔP = pressure drop, pounds per square foot.

g_c = $32.174 \frac{\text{(lb-mass) ft.}}{\text{(lb. force) sec.}^2}$

L = length of core, ft.

ρ = fluid density, lb-mass per cubic foot.

μ' = absolute viscosity, pounds-mass per ft. second.

D_g = diameter of average grain, ft.

V_a = apparent velocity, $\left(\frac{\text{rate of flow}}{\text{cross-sectional area}}\right)$ ft. per second.

A comparison of equation (C-1) to the Fanning equation (B-11) and of equation (C-2) to the Reynolds number equation (B-17), reveals that these equations differ only slightly. These differences result from modifications made by Fancher et al to accommodate the flow of fluids through permeable media rather than through pipes. Their data indicates that the flow of fluids through porous media resembles that through pipe in that there is a change from laminar to turbulent flow as velocities increase. The correlation of f_{FLB} versus N_{ReFLB} results in a straight line relationship with a slope of minus one, on log coordinates, for conditions of laminar flow, regardless of the porous material through which the flow is taking place. At velocities that correspond to a N_{ReFLB} of about 1.0, the relationship between f_{FLB} and

N_{ReFLB} begins to flatten out as the transition to turbulent flow takes place.

Laminar Steady-State Flow

The work of Fancher et al showed that for conditions of laminar flow

$$f_{FLB} = \frac{C_{FLB}}{N_{ReFLB}} \qquad (C-3)$$

where C_{FLB} is a constant characteristic of the porous medium.

Substituting equation (C-2) into (C-3), and the resultant equation into equation (C-1) and solving for V_a

$$V_a = \frac{g_c D_g^{\ 2}}{2C_{FLB}} \quad \frac{\Delta P}{\mu' L} \qquad (C-4)$$

An independent approach to the laminar flow of fluids through porous media is based on the early work of Darcy which has resulted in the so-called Darcy Law, commonly expressed as

$$V_a = \frac{q}{A} = \frac{K}{\mu} \left(- \frac{\Delta P}{\Delta L} \right) \qquad (C-5)$$

where

V_a = apparent velocity under mean flowing conditions, cm. per second.

q = volume flow rate, cu. cm. per second.

A = cross-sectional area of flow, sq. cm.

μ = viscosity of the fluid, cp.

$-\frac{\Delta P}{\Delta L}$ = pressure drop per unit length, atm. per cm.

K = permeability, darcys.

This is the relationship ordinarily employed in petroleum engineering work. The permeability 'K' as defined by equation (C-5) is in fact equal to the value of $g_c D_g^2/2C_{FLB}$ for any particular porous medium.

If the differential form of equation (C-5) is combined with the Gas Law, equation (A-1), applied to a radial system, and if customary field units are introduced

$$Q = \frac{1.4108 \times 10^{-6}\ kh\ P\ dP}{\mu TZ(dr/r)} \tag{C-6}$$

Assuming constant values of k, T, and h, and that the change in pressure is not great so μ may be treated as a constant, equation (C-6) may be partly integrated and written as

$$Q = \frac{1.4108 \times 10^{-6}\ kh}{\mu_a T_a \ \ln(r_f/r_s)} \int_{P_s}^{P_f} \frac{P}{Z}\ dP \tag{C-7}$$

Equation (C-7) may be used in this form with tables of $\int \frac{P}{Z}\ dP$ (49) but where the change in pressure is not great, Z may also be treated as a constant determined at the average pressure condition. In this case equation (C-7) may be integrated and expressed as

$$Q = \frac{0.7054 \times 10^{-6}\ kh\ (P_f^2 - P_s^2)}{\mu_a Z_a T_a\ \ln(r_f/r_s)} \tag{C-8}$$

where

Q = gas flow rate, millions of cubic feet per day at 14.65 psia and 60°F.

k = permeability, millidarcys.

h = pay, ft.

P_f = formation pressure (at exterior drainage boundary), psia.

P_s = flowing sandface pressure, psia.

μ_a = average viscosity, cp.

Z_a = average compressibility factor.

T_a = average flowing temperature, ^{o}R.

r_f = exterior boundary radius, ft.

r_s = effective well bore radius, ft.

Equation (C-8), while sound theoretically is limited in its use even under true laminar conditions because of the inevitable variations in pay and permeability and because of the difficulties associated in determining r_f and r_s.

The following example illustrates the use of the steady-state laminar flow equation.

Example C-1

4.915 MMcfd of 0.6 gravity gas is being injected into a well. The formation is 20 feet thick, the permeability is 50 md., and the temperature of the flowing gas is 640^{o}R. If the injection pressure is 2300 psia at the sandface of a four inch well bore, calculate the pressure 40 feet out from the centre of the well bore.

Solution

$$\left.\begin{aligned} P_c &= 672 \text{ psia} \\ T_c &= 358^{o}\text{R} \end{aligned}\right\} \qquad \text{(Figure A-1)}$$

Assume laminar steady-state flow, and use equation (C-8) and a trial and error procedure.

First trial

Assume $P_f = P_s = 2300$ psia

P_a = 2300 psia

$$P_{r_a} = \frac{2300}{673} = 3.42$$

$$T_{r_a} = \frac{640}{358} = 1.79$$

$$Z_a = 0.887$$ (Figure A-2)

$$\mu_a = 0.0125\ (1.35) = 0.0169$$ (Figure A-10 and A-11)

Substituting into equation (C-8)

$$4.915 = -\frac{(0.7054 \times 10^{-6})(20)(50)(P_f^{\,2} - 2300^2)}{(0.0169)(0.887)(640)\ \ln(40/0.33)}$$

$$P_f^{\,2} = 5{,}290{,}000 - 320{,}100$$

$$P_f = 2230 \text{ psia}$$

Second trial

Assume P_f = 2230 psia

$$P_a = 2265 \text{ psia}$$

$$P_{r_a} = \frac{2265}{673} = 3.37$$

$$Z_a = 0.889$$ (Figure A-2)

$$\mu_a = 0.0125\ (1.33) = 0.0166$$ (Figure A-10 and A-11)

Substituting into equation (C-8)

$$4.915 = -\frac{(0.7054 \times 10^{-6})(20)(50)(P_f^{\,2} - 2300^2)}{(0.0166)(0.889)(640)\ \ln(40/0.33)}$$

$$P_f^{\,2} = 5{,}290{,}000 - 315{,}800$$

$$P_f = 2230 \text{ psia}$$

Hence the answer converges to 2230 psia in two trials.

A similar approach may be used to determine the permeability-pay product where the sandface pressure and the pressure at some distance out in the reservoir are known at a particular flow rate.

Turbulent Steady-State Flow

During the production of gas from a reservoir, the linear velocity and the corresponding Reynolds number of the gas increases as it approaches the well bore. Laminar conditions will prevail throughout the reservoir if the linear velocity of gas at the sandface is such that the corresponding modified (Fancher, Lewis and Barnes) Reynolds number is less than approximately one. For higher flow rates, turbulent conditions will exist at the sandface, and will prevail some distance back into the reservoir. This distance will depend upon the velocity of the gas. Thus, the common situation arises where turbulent conditions prevail for some distance out from the well bore, and laminar conditions exist from this intermediate point to the boundaries of the reservoir.

Forscheimer (30) has proposed that flow under both laminar and turbulent conditions may be handled by a quadratic equation including two components; the pressure gradients due to the viscous shearing forces and to the inertial forces. The Forscheimer equation (in absolute units as used in equation (C-5)), may be expressed as

$$-\frac{\Delta P}{\Delta L} = \frac{\mu V_a}{K} + \beta \rho V_a^{2} \qquad \text{(C-9)}$$

where

β = a constant of proportionality referred to as the turbulence factor, cm^{-1}.

The meaning of β can be better illustrated if we consider the case of a well producing from the centre of a radial reservoir of a size such that turbulent flow exists near the well bore. The total pressure drop from the outer boundary to the well bore may be expressed as

$$\Delta P_T = \Delta P_L + \Delta P_{TT} \qquad \text{(C-10)}$$

where

ΔP_T = total pressure drop.

ΔP_L = pressure drop in region of laminar flow.

ΔP_{TT} = pressure drop in regions of transitional and turbulent flow.

If we further define:

L_L = length of regions of laminar flow.

L_{TT} = length of region of transitional and turbulent flow.

Then a friction factor for the turbulent-transitional region may be expressed by a relationship similar to equation (C-1)

$$f_{TT} = \frac{g_c D_g \Delta P_{TT}}{2 V_a^2 \rho \Delta L_{TT}} \tag{C-11}$$

then by combining equations (C-4), (C-11) and (C-10),

$$\Delta P_T = \frac{2C_{FLB}}{g_c D_g^2} \mu' V_a \Delta L_L + \frac{2\ f_{TT}}{g_c D_g}\ V_a^2 \Delta L_{TT} \tag{C-12}$$

this equation is identical to the Forscheimer equation (C-9) where

$$\frac{2\ 'C_{FLB} \Delta L_L}{g_c D_g^2 \Delta L} = \frac{\mu}{K} \tag{C-13}$$

and

$$\frac{2\ f_{TT}}{g_c D_g}\ \frac{\Delta L_{TT}}{\Delta L} = \beta \tag{C-14}$$

The turbulence factor is analogous to a friction factor divided by a grain diameter.

If the differential form of equation (C-9) is combined with the Gas Law, equation (A-1) and is applied to a radial system, expressed in

customary field units, and integrated assuming constant values of k, T, μ, h, Q and Z, then it may be expressed as

$$P_f^2 - P_s^2 = \frac{1.418 \times 10^6}{kh} \mu_a Z_a T_a Q \ln\left(\frac{r_f}{r_s}\right) + \frac{3.140}{h^2} \times 10^{-6} \beta Z_a T_a Q^2 \left(\frac{1}{r_s} - \frac{1}{r_f}\right) \qquad \text{(C-15)}$$

where

P_f = formation pressure, psia.

P_s = flowing sandface pressure, psia.

k = permeability, md.

h = pay, ft.

μ_a = average viscosity, cp.

T_a = average temperature, °R.

Z_a = average compressibility factor.

r_f = exterior boundary radius, ft.

r_s = effective well bore radius, ft.

β = turbulence factor.

Q = gas flow rate, millions of cubic feet per day at 14.65 psia and 60°F.

Before this equation may be used, the turbulence factor 'β' must be evaluated. Janicek and Katz (45) have made measurements of the turbulence factor for cores of various permeabilities and porosities and have correlated their data in the form of a plot of turbulence factor versus permeability with porosity as a parameter. Cornell and Katz (17) have reviewed these data and proposed a simpler relationship of the turbulence factor to the permeability itself. They found that for dolomites, limestones and sandstones an approximate relationship exists between the turbulence factor and the permeability. This relationship may

be expressed as

$$\beta = \frac{4.11 \times 10^{10}}{k^{4/3}} \qquad \text{(C-16)}$$

where

k = permeability, millidarcys.

Cornell and Katz have correlated pressure drop data on consolidated and unconsolidated cores, in terms of a modified friction factor, f_{CK}, and a modified Reynolds number, N_{ReCK}. These factors are really very similar to those employed by Fancher et al, and are defined as

$$f_{CK} = \frac{64\ g_c\ \Delta P}{\beta\ V_a^2\ \rho L} \qquad \text{(C-17)}$$

and

$$N_{ReCK} = \frac{\beta K}{6.33 \times 10^{10}}\ \frac{V_a \rho}{\mu} \qquad \text{(C-18)}$$

Combining equations (C-1) and (C-17), it is apparent that the friction factor defined by Fancher et al and by Cornell and Katz are related through

$$\frac{2f_{FLB}}{D_g} = \frac{\beta f_{CK}}{64}$$

and combining equations (C-2) and (C-18), it is clear that the D_g/μ' in the Fancher relationship is replaced by $\beta K/6.33 \times 10^{10}\mu$ in the Cornell-Katz modified Reynolds number.

If the value of 'β' given by equation (C-16) is incorporated into equation (C-18), for a radial system, and converted to field units, the modified Reynolds number at the sandface becomes

$$N_{ReCK} = \frac{0.09101\ GQ}{r_s h\ \mu\ k^{1/3}} \qquad \text{(C-19)}$$

where

P = pressure, psia.

L = length of flow, ft.

G = specific gravity of gas.

Q = flow rate, millions of cubic feet per day at 14.65 psia and 60°F.

k = permeability, millidarcys.

μ = fluid viscosity, centepoises.

h = pay, ft.

r_s = effective well bore radius, ft.

The Cornell and Katz friction factor plot (17) (49) indicates that the change from laminar to turbulent flow for all types of porous media begins at an N_{ReCK} of about 0.1. The transition region where the flow is partly turbulent corresponds to the N_{ReCK} range of from 0.1 to about 1.0.

The Turbulence Contribution Factor

Equation (C-15) for laminar-turbulent radial flow may be expressed in the form of a quadratic equation

$$P_f^2 - P_s^2 = aQ + bQ^2 \qquad \text{(C-20)}$$

where

$$a = \frac{1.418 \times 10^6 \mu_a Z_a T_a \ln (r_f/r_s)}{kh}$$

and

$$b = \frac{3.14 \times 10^{-6} \beta G Z_a T_a}{h^2}\left(\frac{1}{r_s} - \frac{1}{r_f}\right)$$

Govier (35) has shown that if a turbulence contribution factor 'F_t' is defined as

$$F_t = 1 + \frac{b}{a} Q \qquad \text{(C-21)}$$

then the laminar-turbulent flow equation may be written as

$$P_f^2 - P_s^2 = a\, Q\, F_t \qquad \text{(C-22)}$$

and the turbulence contribution factor 'F_t' may be evaluated by combining

equation (C-15) and the emperical relationship between β and k, equation (C-16).

If this is done and the small value of $1/r_f$ is neglected

$$F_t = 1 + \frac{0.09101\ GQ}{r_s h k^{1/3}\ \mu_a} \ \frac{1}{\ln(r_f/r_s)} \tag{C-23}$$

It may be further shown that for the common case where the drainage radius is of the order of one-half mile and the $\ln(r_f/r_s)$ is of the order of 9, equation (C-23) reduces to

$$F_t \simeq 1 + 0.01 \frac{GQ}{r_s h \mu_a k^{1/3}} \tag{C-24}$$

or

$$F_t \simeq 1 + 0.1\ (N_{ReCK}) \tag{C-25}$$

The solution of equation (C-24) is presented in graphical form as Figure C-1.

At a Reynolds number of 0.1, where Cornell and Katz found that the change from laminar to turbulent flow begins, equation (C-25) shows that the turbulence contribution factor would be approximately 1.01.

To solve the laminar-turbulent equation (C-15) or its equivalent (C-22), the turbulence factor can be evaluated from equation (C-23) or for certain drainage conditions from Figure C-1. It should be emphasized that neither the equation nor the figure are precise, since they incorporate the emperical correlation between the turbulence factor 'β' and the reservoir permeability 'k' as developed by Cornell and Katz.

The following example illustrates the use of the turbulence contribution chart, Figure C-1, in the calculation of sandface pressures.

Example C-2

5.0 MMcfd of 0.75 gravity gas is being produced from a well completed in a formation with a 20 foot pay section and a permeability of

60 md. The shut-in reservoir pressure is 2500 psia, the well bore radius is 4 inches, and the average reservoir temperature is 660°R. A calculation has revealed that the average compressibility factor is 0.80 and the viscosity is 0.02 cp.

Calculate the sandface pressure assuming that the steady-state radius of drainage is one-half mile.

$$\frac{GQ}{\mu_a r_s h} = \frac{(0.75)(5.0)}{(0.02)(0.33)(20)} = 28.4$$

From Figure C-1

$$F_t = 1.073$$

Substituting into equation (C-22)

$$P_s^{\ 2} = 2500^2 - \frac{(1.073)(1.418)(10^6)(0.02)(0.80)(660)(5)(8.989)}{(20)(60)}$$

$$= 2500^2 - 601{,}800$$

$$P_s = 2377 \text{ psia}$$

Direct use of the laminar flow equation for this problem gives a sandface pressure of 2385 psia.

B. UNSTEADY-STATE FLOW IN THE RESERVOIR

Flow in a porous medium is defined as unsteady-state when the mass flow rate into an element of the medium is not the same as the mass flow rate out of that element. Hence the fluid content of the element changes with time. Such changes in the amount of fluid in the porous medium are possible because of the compressibility of fluids. The variables in unsteady-state flow, in addition to those already used for steady-state flow, are time, the porosity of the formation and the compressibility of the fluid.

The basic approach to the solution of an unsteady-state problem,

must be consideration of the system as a separate entity. A material balance equation known as the continuity equation, is then written to account for the fluid injected into and produced out of the system. When this equation is combined with the Gas Law (the equation of state) and Darcy's Law (the equation of motion), it provides a method of solution of unsteady-state problems for known boundary conditions.

For the radial flow case, the equation of continuity may be written as follows

$$\frac{\partial\ (\rho V)}{\partial\ r} + \frac{\rho\ V}{r} = -\ \phi\ \frac{\partial \rho}{\partial t} \qquad \text{(C-26)}$$

where

V = velocity in the radial direction.

r = radius of the volume being considered.

ϕ = porosity of the volume being considered.

ρ = fluid density.

t = time.

Equation (C-5), Darcy's Law for radial flow may be written as

$$V = -\ \frac{K}{\mu}\ \frac{\partial P}{\partial r} \qquad \text{(C-27)}$$

and the Gas Law, equation (A-1) may be written as

$$\rho = \frac{MP}{ZRT} \qquad \text{(C-28)}$$

Combining equations (C-26), (C-27) and (C-28) and the assumption that viscosity, compressibility factor and permeability are constant, the following equation may be written for laminar,

radial flow in a horizontal plane.

$$\frac{\partial^2 P^2}{\partial r^2} + \frac{1}{r}\,\frac{\partial P^2}{\partial r} = \frac{\mu\phi}{KP}\,\frac{\partial P^2}{\partial t} \qquad (C\text{-}29)$$

It should be pointed out that equation (C-29) is not the rigorous differential equation, but is only approximate in that the right-hand side should be $\frac{\mu\phi}{K}\,\frac{\partial P}{\partial t}$. This approximation is not serious unless drawdowns are very large.

Equation (C-29) is non-linear (the dependent variable P^2 appears to powers other than one) and insoluble as written. If the 'P' in $\mu\phi/KP$ is assumed to be constant at a value of the average pressure, P_a, then the equation is linear and can be solved by several methods. These methods include the analytical solutions for constant production rate by Hurst and Van Everdingen (42). Cornell (20)(21) has applied the Hurst and Van Everdingen solution to constant rate gas flow problems. Cornell and Katz (17) have adapted the Schmidt graphical method used for unsteady-state heat flow problems to the solution of the linearized equation. This method is particularly useful in visualizing transient pressure gradients.

One of the most convenient approaches to solving unsteady-state flow problems is through definition of certain dimensionless quantities and solution of the linear equation in normalized form.

If the following dimensionless quantities are defined:

dimensionless pressure $$P_D = \left(\frac{P}{P_f}\right)^2 \qquad (C\text{-}30)$$

dimensionless radius $$r_D = \frac{r}{r_s} \qquad (C\text{-}31)$$

dimensionless time $$t_D = \frac{2.634 \times 10^{-4}\, kP_a t}{\mu_a \phi r_s^2} \qquad (C\text{-}32)$$

dimensionless flow rate $$q_D = \frac{1.418 \times 10^6 \mu_a Z_a T_a Q}{hk\, P_f^2} \qquad (C\text{-}33)$$

dimensionless drawdown $\quad P_t = \frac{1}{q_D}\left(\frac{P_f^2 - P^2_{(r,t)}}{P_f^2}\right)$ (C-34)

where

P = pressure, psia.

r = radius, ft.

k = permeability, md.

t = time, hours.

μ_a = average viscosity, cp.

Z_a = average compressibility factor.

ϕ = gas filled porosity, fraction.

T_a = average temperature, °R.

Q = flow rate, millions of cubic feet per day at 14.65 psia and 60°F.

h = pay, ft.

$P_{(r,t)}$ = pressure at any radius, r, after well has been produced at constant rate, q_D, for time, t.

then equation (C-29) may be normalized (made dimensionless), by dividing the numerators by P_f^2 and the denominators by r_s^2, and expressed as

$$\frac{\partial 2P_D}{\partial r_D^2} + \frac{1}{r_D}\frac{\partial P_D}{\partial r_D} = \frac{\partial P_D}{\partial t_D} \qquad \text{(C-35)}$$

and generalized solutions in terms of the defined ratios may be developed for various boundary conditions.

Hurst and Van Everdingen (44) have solved this equation for a constant flow rate "Q" and for an infinite reservoir, and have shown that for t_D greater than 100,

$$P_t = 1/2\ (\ln t_D + 0.80907) \qquad \text{(C-36)}$$

They also solved the equation for a constant flow rate from a finite

reservoir with a closed circular outer boundary, and have shown that

$$P_t = \frac{0.5 + 2t_D}{r_D^2 - 1} - \frac{3r_D^4 - 4r_D^4 (\ln r_D) - 2r_D^2 - 1}{4(r_D^2 - 1)^2} \quad \text{(C-37)}$$

for values of t_D that correspond to times when the boundary effects are felt. Hurst and Van Everdingen also solved the equation for the case of a finite reservoir with a constant pressure at the outer boundary, but the solution is complex and for this reason is not reproduced here in equation form.

These and other solutions may be expressed in tabular or graphical form, the latter being perhaps the more convenient for general use. Katz et al (49) have presented such graphical representations of the solutions for certain boundary conditions. Aziz and Flock (4) have recently published graphical representations of the solutions of the unsteady-state flow equation for the constant production rate case. Their graphical presentation is especially convenient because it combines the solution for the boundary conditions of greatest interest in a single plot. The form of plot is produced as Figure C-2, and includes solutions for the following boundary conditions.

1. Infinite radial reservoir.
2. Finite reservoir, sealed outer boundary.
3. Finite reservoir, constant pressure at the outer boundary.

The following examples illustrate the use of this figure.

Example C-3

A gas well located in a very large reservoir which has been shut in for a lengthy period, is produced for five hours at the rate of 3.01 MMcfd. Calculate the sandface pressure at this time if the following reservoir data is given.

P_f = 2300 psia

$T = 640°R.$

$\phi = 0.15$

$k = 28$ md.

$h = 12$ ft.

$r_s = 0.5$ ft.

The sandface pressure must be assumed to calculate Z_a and μ_a. However, for simplification this portion of the calculation will be left out of the trial and error procedure. It will be assumed that the following values are known.

$Z_a = 0.89$

$\mu_a = 0.0168$

Solution (For five hours producing from a large reservoir, this can be considered as an infinite reservoir.)

First trial

Assume $P_s = 2100$ psia

$P_a = 2200$ psia

From equation (C-32)

$$t_D = \frac{2.634 \times 10^{-4}\ (28)(2200)(5)}{(0.0168)(0.15)(0.5)^2} = 128{,}800$$

so from Figure C-2 (infinite case)

$P_t = 6.28$

In this case P_t may also be calculated from equation (C-36).

From equation (C-33)

$$q_D = \frac{1.418 \times 10^6\ (0.0168)(0.89)(640)(3.01)}{(12)(28)(2300)^2} = 0.0230$$

Substituting into equation (C-34)

$$6.28 = \frac{1}{0.0230}\left(\frac{2300^2 - P_s^{\ 2}}{2300^2}\right)$$

$P_s = 2127$ psia

Second trial

Assume $P_s = 2127$ psia

$$P_a = \frac{2127 + 2300}{2} = 2214 \text{ psia}$$

$$t_D = \frac{(2.634)(10^{-4})(28)(2214)(5)}{(0.0168)(0.15)(0.25)} = 129{,}600$$

From Figure C-2

$$P_t = 6.29$$

$$q_D = \frac{(1.418)(10^6)(0.0168)(0.89)(640)(3.01)}{(12)(28)(2300)^2} = 0.0230$$

Once again solving equation (C-34)

$$P_s = 2127 \text{ psia}$$

Hence the sandface pressure converges to 2127 psia in two trials.

Example C-4

Assume that the well in Example C-3 is located in a radial reservoir only 2000 feet in diameter, and is produced at 3.01 MMcfd for 100 hours. Reservoir data are:

P_f = 2300 psia

T = 640°R.

φ = 0.15

k = 28 md.

h = 12 ft.

r_s = 0.5 ft.

Z_a = 0.89

μ_a = 0.0168 cp.

Solution (This is the case of a finite reservoir with a sealed outer boundary.)

$$r_D = \frac{r}{r_s} = \frac{1000}{0.5} = 2000$$

To avoid numerous trials, we will assume a sandface pressure we know to be approximately correct.

Assume P_s = 2075 psia

$$P_a = \frac{2300 + 2075}{2} = 2187 \text{ psia}$$

From equation (C-32)

$$t_D = \frac{2.634 \times 10^{-4}\ (28)(2187)(100)}{(0.0168)(0.15)(0.5)^2} = 2{,}560{,}000$$

so from Figure C-2 (finite case-sealed outer boundary)

$$P_t = 8.1$$

From equation (C-33)

$$q_D = \frac{1.418 \times 10^6\ (0.0168)(0.89)(640)(3.01)}{(12)(28)(2300)^2} = 0.0230$$

Substituting into equation (C-34)

$$8.1 = \frac{1}{0.0230}\left(\frac{2300^2 - P_s^{\ 2}}{2300^2}\right)$$

$$P_s = 2075 \text{ psia}$$

So the assumed sandface pressure is correct and no further trial is necessary.

The problem involving the finite reservoir-constant pressure at the outer boundary is handled in the same manner as Example C-4, except that the appropriate portion of Figure C-2 is used to determine P_t. This approach may also be used to calculate the permeability-pay product when the sandface pressure and a pressure at some distance out in the reservoir are known for a particular flow rate. It may also be used to estimate the

time to reach a certain pressure, or the degree of stabilization. The latter matter is dealt with at length in Appendix D.

In the case where the rate of flow from a reservoir has been changed, multiple transients result in the reservoir. This is known as the flow after flow case, and the pressure distribution at any time may be calculated. The approach involves determining the pressure change associated with each change of flow rate, and summing these pressure changes for the time over which each change was in effect. The calculation may be made through use of the following equation.

$$\frac{P_f^{\,2} - P^2_{(r,t)}}{P_f^{\,2}} = [q_{D_1} P_{t_1} + (q_{D_2} - q_{D_1}) P_{t_2} + (q_{D_3} - q_{D_2}) P_{t_3} + \ldots + (q_{D_n} - q_{D_{(n-1)}}) P_{t_n}] \qquad \text{(C-38)}$$

where

t_1 = total elapsed time since the first flow rate began.

t_2 = total elapsed time since the second flow rate began.

t_n = total elapsed time since nth flow rate began.

q_{D_1} = first dimensionless flow rate.

q_{D_n} = nth dimensionless flow rate.

P_{t_1} = dimensionless pressure change number evaluated at time t_1.

P_{t_n} = dimensionless pressure change number evaluated at time t_n.

The following example illustrates the application of equation (C-38).

Example C-5

A gas well which has been shut in for a lengthy period, is opened and produced at a rate of 3.01 MMcfd for 4.0 hours. The rate is increased to 6.02 MMcfd and maintained at this rate for another 4.0 hours. Calculate the sandface pressure at the end of the second production period, assuming an infinite reservoir.

Reservoir data are:

$P_f = 2300$ psia

$\mu_a = 0.0168$ cp.

$h = 12$ ft.

$Z_a = 0.89$

$T_f = 640°R.$

$k = 28$ md.

$r_s = 0.5$ ft.

$\phi = 0.15$

Solution

First trial

Assume $P_s = 2000$ psia

$P_a = 2150$ psia

$$t_{D_1} = \frac{(2.634)(10^{-4})(28)(2150)(8)}{(0.0168)(0.15)(0.25)} = 201,400$$

$$t_{D_2} = \frac{(2.634)(10^{-4})(28)(2150)(4)}{(0.0168)(0.15)(0.25)} = 100,700$$

From Figure C-2

$P_{t_1} = 6.49$

$P_{t_2} = 6.14$

$$q_{D_1} = \frac{(1.418)(10^6)(0.89)(0.0168)(640)(3.01)}{(12)(28)(2300)^2} = 0.0230$$

$$q_{D_2} = \frac{(1.418)(10^6)(0.89)(0.0168)(640)(6.02)}{(12)(28)(2300)^2} = 0.0460$$

Substituting into equation (C-38)

$$\frac{2300^2 - P_s^{\ 2}}{2300^2} = [(0.0230)(6.49) + (0.0230)(6.14)]$$

$$P_s = 1937 \text{ psia}$$

Second trial

Assume P_s = 1937 psia

$$P_{\bar{s}} = \frac{1937 + 2300}{2} = 2119 \text{ psia}$$

$$t_{D_1} = \frac{(2.634)(10^{-4})(28)(2119)(8)}{(0.0168)(0.15)(0.25)} = 198{,}500$$

$$t_{D_2} = \frac{(2.634)(10^{-4})(28)(2119)(4)}{(0.0168)(0.15)(0.25)} = 99{,}230$$

From Figure C-2

$$P_{t_1} = 6.49$$

$$P_{t_2} = 6.14$$

$$q_{D_1} = \frac{(1.418)(10^6)(0.0168)(0.89)(640)(3.01)}{(12)(28)(2300)^2} = 0.0230$$

and

$$q_{D_2} = 0.0460$$

Substituting into equation (C-38)

$$\frac{2300^2 - P_s^{\ 2}}{2300^2} = [(0.0230)(6.49) + (0.0230)(6.14)]$$

$P_s = 1937$ psia

Hence the answer converges to 1937 psia in two trials.

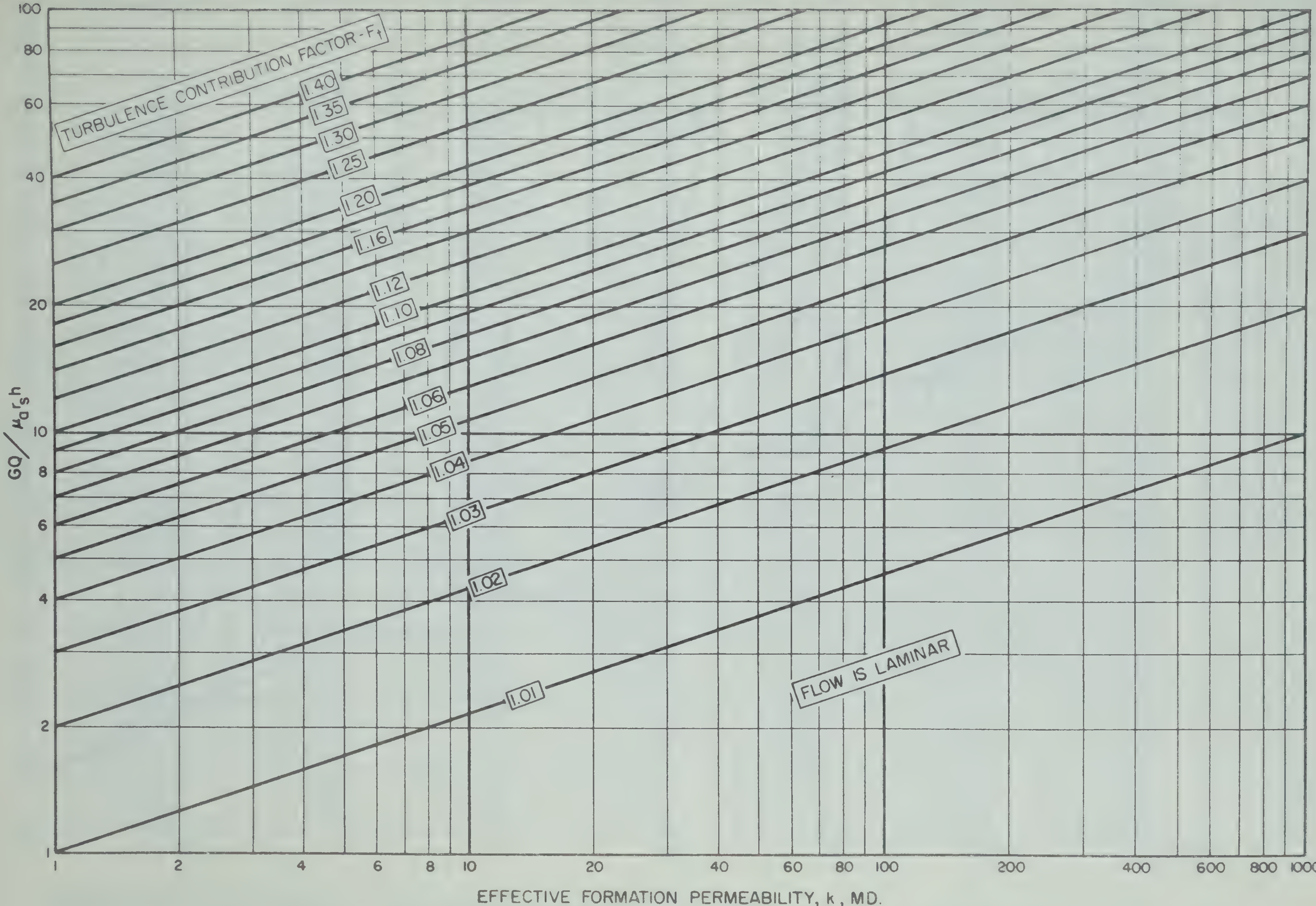


FIGURE C-1 — TURBULENCE CONTRIBUTION FACTORS.
(Redrawn from Govier,(35).

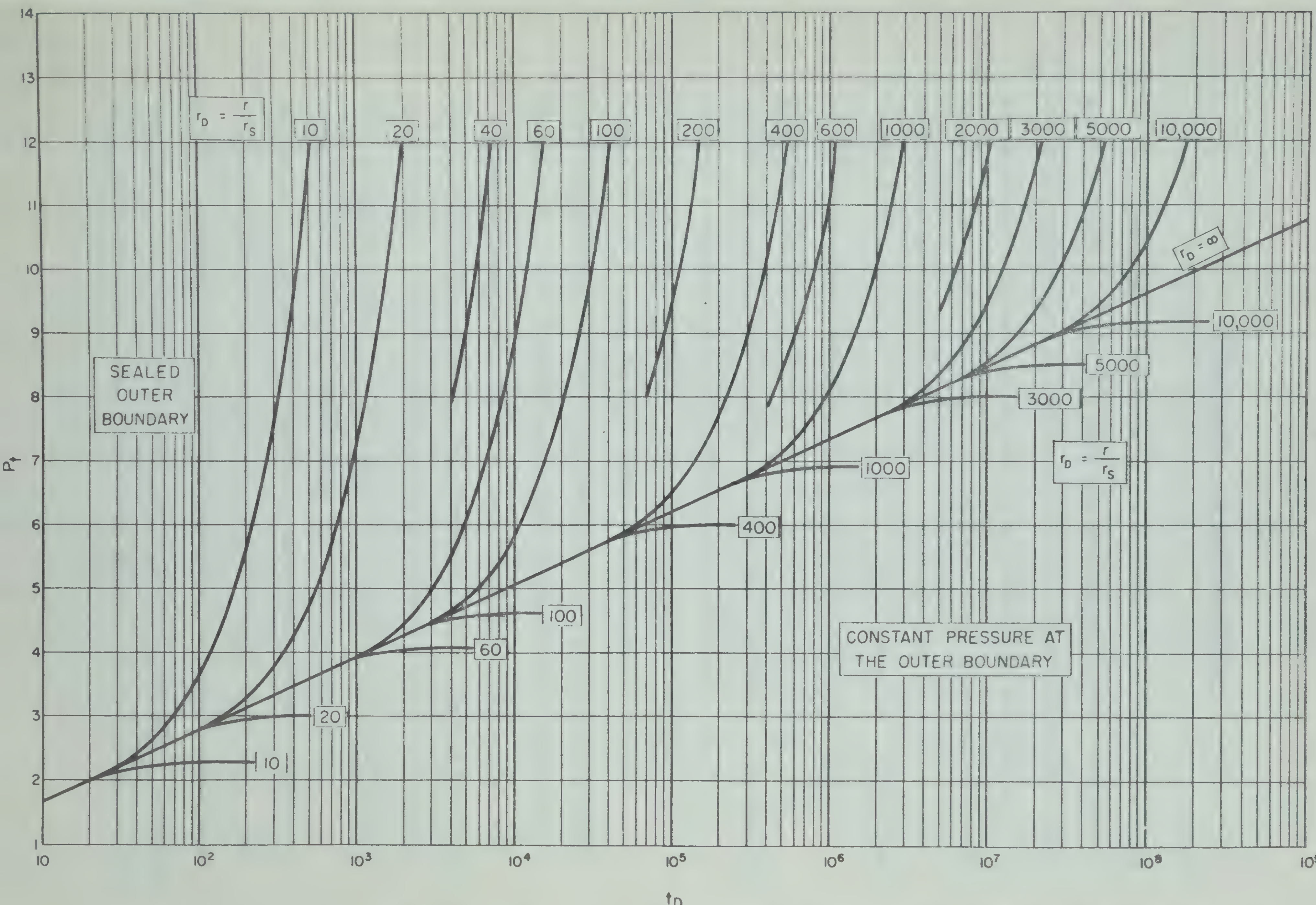


FIGURE C-2 — SOLUTIONS OF THE UNSTEADY-STATE FLOW EQUATION FOR THE CONSTANT PRODUCTION RATE CASE.

(Replotted from Aziz and Flock, 4)

APPENDIX D

STABILIZATION AND RELATED MATTERS

The simplicity of form of the equations describing the steady-state radial flow of gas in a reservoir as compared to their unsteady-state counterpart, has long suggested that well test data obtained under steady-state conditions may be more easily interpreted than similar unsteady-state data. However, it is clear that the production of a well in a finite reservoir is inherently an unsteady-state process, and that steady-state analysis of the situation is an approximation. Nonetheless, consideration of the unsteady-state relationship, particularly as the graphical solution of Cornell and Katz, indicates that at any instant of time the steady-state equation is satisfied if movement of the gas is assumed to originate at the apparent radius of drainage - i.e. at the radius obtained by extrapolating the linear portion of the $P_f^2 - P_s^2$ versus ln (r/r_s) curve, to P_f^2. This means that data points taken at times corresponding with equal apparent radii of drainage would simulate steady-state data. This approach prompts one to think of a condition of apparent stabilization for which the steady-state equations are adequately descriptive. Clearly, this condition must be close to that which would be reached after an appropriately long time, and must be changing very slowly with time. The time required to reach such a condition depends upon the arbitrary definition of the condition. A logical definition is that when the transient pressure-radius profile first reaches the edge of the spacing unit in which the well is drilled. For a fully developed field, this is the point where the profile would intersect that of the neighboring wells. For a field not completely developed, while the profile would eventually extend further, its restriction to the

edge of the spacing unit simulates full development. Janicek and Katz (45) have shown that the ratio of the apparent radius of drainage (as here defined) to the effective radius (to the edge of the spacing unit), is approximately 0.60. Thus, the time to attain this arbitrarily defined stabilization is that time required for the apparent radius of drainage to move six-tenths of the distance towards the edge of the pattern. This time may be calculated from unsteady-state theory.

The equation for steady-state laminar radial flow of a gas may be expressed as

$$q_D = \frac{1}{\ln(r_a/r_s)} \frac{(P_f^2 - P_s^2)}{P_f^2} \qquad \text{(D-1)}$$

where

q_D = the dimensionless flow rate.

P_f = formation pressure, psia.

P_s = flowing sandface pressure, psia.

r_s = effective well bore radius, ft.

r_a = the apparent radius of drainage, ft. (which is equal to 0.60 r_f at the time the transient pressure-radius profile reaches the edge of the drainage pattern.)

If the equation is combined with the unsteady-state flow equation in the form of equation (C-34)

$$P_t = \frac{1}{q_D} \frac{P_f^2 - P^2_{(r,t)}}{P_f^2} \qquad \text{(C-34)}$$

the following relationship which is valid for the condition of apparent stabilization results

$$P_t = \ln (r_a/r_s) \frac{P_f^{\,2} - P^2_{(r,t)}}{P_f^{\,2} - P_s^{\,2}} \qquad (D-2)$$

and if the unsteady-state flow equation is written in terms of the sandface, equation (D-2) may be written as

$$P_t = \ln (r_a/r_s) \qquad (D-3)$$

The dimensionless time necessary to attain this condition can be determined from Figure C-2, for the appropriate boundary conditions. The actual time corresponding with this time, may be determined through equation (D-4)

$$t_D = \frac{2.634 \times 10^{-4} \, kP_a t_s}{\mu_a \, \phi r_s^{\,2}} \qquad (D-4)$$

where

k = permeability, md.

P_a = average pressure, psia.

t_s = the time of stabilization, hours.

μ_a = average viscosity, cp.

ϕ = gas filled porosity, fraction.

r_s = effective well bore radius, ft.

This determination requires a knowledge of the average pressure, which in turn requires that the sandface pressure is known. The sandface pressure 'P_s' may be determined from a knowledge of the apparent radius of drainage, which coincides with the arbitrarily selected definition of stabilization, and the steady-state equation (C-8).

$$Q = \frac{0.7054 \times 10^{-6} \, kh \, (P_f^{\,2} - P_s^{\,2})}{\mu_a Z_a T_a \, \ln(r_a/r_s)} \qquad (C-8)$$

where

Q = gas flow rate, millions of cubic feet per day at 14.65 psia and 60°F.

k = permeability, md.

h = pay, ft.

P_f = formation pressure, psia.

P_s = flowing sandface pressure, psia.

μ_a = average viscosity, cp.

Z_a = average compressibility factor.

T_a = average temperature, ^oR.

r_a = apparent radius of drainage, ft.

r_s = effective well bore radius, ft.

The above outlined approach may be simplified by development of an equation which enables one to estimate directly the time of stabilization. In the case of a finite reservoir with a constant pressure at the outer boundary, Figure C-2 (solution of the unsteady-state flow equation) shows that the dimensionless pressure drawdown, P_t, becomes constant at some t_D, for each dimensionless radius, r_D. This particular portion of the figure was taken from work of Hurst et al (44), and the relationship between 'r_D' and 't_D' at the instant when 'P_t' becomes constant may be expressed as

$$r_D = 2.6408\ t_D^{0.48858} \qquad \text{(D-5)}$$

which may be approximated as

$$r_D = \text{"constant"}\ t_D^{0.5} \qquad \text{(D-6)}$$

where the constant varies depending on the value of t_D. If equation (D-6) is combined which equation (D-4), the following results

$$t_s = \text{"constant"}\ \frac{r_f^2\ \mu_a\ \phi}{k\ P_a} \qquad \text{(D-7)}$$

where the constant varies from some 500 to about 900 over the range of dimensionless times normally encountered.

For the infinite reservoir case, Janicek and Katz (45) have expressed the relationship between 'r_D' and 't_D' at the time 'P_t' becomes constant as

$$r_D = 2\ t_D^{0.5} \qquad \text{(D-8)}$$

This expression results in the following equation

$$t_s = 948\ \frac{r_f^2\ \mu a\ \phi}{k\ P_a} \qquad \text{(D-9)}$$

Muskat (63) has developed a method for calculating the time of stabilization for a radial water-drive oil reservoir. When applied to a finite radial gas reservoir with a fixed outer boundary, the approach results in the following relationship

$$t_s = 950\ \frac{r_f^2\ \mu a\ \phi}{k\ P_a} \qquad \text{(D-10)}$$

Another criterion of stabilization that is often used is the time when the sandface pressure is dropping at a specified low rate per unit of time.

For a known constant production rate, the pressure drop after any time may be measured from the slope of the appropriate 'P_t' versus 't_D' curve, in Figure C-2. From equations (C-34) and (C-32) this rate of pressure decline may also be expressed as

$$\frac{dP_t}{dt_D} = \frac{d\left(\frac{1}{q_D}\ \frac{P_f^2 - P_s^2}{P_f^2}\right)}{d\left(\frac{2.634 \times 10^{-4}\ k\ P_a t}{\mu a\ \phi\ r_s^2}\right)} \qquad \text{(D-11)}$$

If we combine equations (D-11) and (C-33) and assume that μ_a, Z_a, P_a, Q, k, T_a, and ϕ, are constants, and that $P_a = P_s$ (this is a reasonable approximation where the drawdown is not large the following equation results.

$$\frac{dP_t}{dt_D} = \frac{5.356 \times 10^{-3}\ \phi r_s^{\ 2} h}{Z_a T_a Q}\left(-\ \frac{dP_s}{dt}\right) \qquad \text{(D-12)}$$

where

$-\dfrac{dP_s}{dt}$ = the pressure decline at the sand-face, psi per hour.

In the most commonly occurring case, that of the finite reservoir with a sealed outer boundary, Van Everdingen and Hurst (44) have shown that

$$P_t = \frac{0.5 + 2t_D}{r_D^{\ 2} - 1} - \frac{3r_D^{\ 4} - 4r_D^{\ 4}\ (\ln r_D) - 2\ r_D^{\ 2} - 1}{4(r_D^{\ 2} - 1)^2} \qquad \text{(C-37)}$$

so

$$\frac{dP_t}{dt_D} = \frac{2}{r_D^{\ 2} - 1} \qquad \text{(D-13)}$$

or since $r_D^{\ 2} >> 1$

$$\frac{dP_t}{dt_D} = \frac{2}{r_D^{\ 2}} = 2\left(\frac{r_s}{r_f}\right)^2 \qquad \text{(D-14)}$$

Combining equations (D-12) and (D-14)

$$2\left(\frac{r_s}{r_f}\right)^2 = \frac{5.356 \times 10^{-3}\ \phi r_s^{\ 2}\ h}{Z_a T_a Q}\left(-\ \frac{dP_s}{dt}\right) \qquad \text{(D-15)}$$

Equation (D-15) shows that when the pressure transient reaches the edge of the drainage area, the rate of pressure decline will depend upon the flow rate, and the reservoir and gas properties. If stabilization is defined as a specific rate of pressure decline, the "stabilized radius of drainage" calculated from the equation will vary, not only from well to well, but for a particular well as the flow rate changes. A different (dP_s/dt) would be required for each flow rate, to ensure equivalent stabilization. Since this

situation is undesirable, the rate of pressure decline method for defining stabilization is rejected, and an approach more closely related to a set radius of drainage is recommended.

Considering equations (D-7) based on work of Hurst, (D-9) based on work of Janicek and Katz, (D-10) based on work of Muskat, and several equations by others as summarized by Van Poolen (91), it is concluded that the following equation should give a reasonable estimate of the time to stabilization. It should be noted that the constant is rounded to 1000, and that the reservoir pressure is used in place of the average pressure in equations (D-7), (D-9) and (D-10).

$$t_s = 1000 \frac{r_f^2 \mu_a \phi}{k P_f} \qquad \text{(D-16)}$$

where

t_s = time of stabilization, hours.

r_f = radius to the exterior boundary, ft.

μ_a = average viscosity, cp.

ϕ = gas filled porosity, fraction.

k = permeability, md.

P_f = formation pressure, psia.

The following examples illustrate the use of this equation for calculating the time to stabilization, and also show that the rate of pressure decline method is not a useful approach for estimating times to stabilization.

<u>Example D-1</u>

Calculate the time of stabilization for a low permeability, low pressure well, given the following reservoir data.

P_f = 625 psia.

T_a = 520°R.

$k = 10$ md.

$\phi = 0.20$

$h = 20$ ft.

$G = 0.57$

$T_c = 341°R.$

$P_c = 665$ psia.

$r_s = 0.25$ ft.

$r_f = 2640$ ft. (1 section spacing)

$Z_a = 0.90$ (at an estimated average reservoir pressure of 550 psia.)

$\mu_a = 0.011$ cp. (at an estimated average reservoir pressure of 550 psia.)

The well is flowed at a constant flow rate of 500 Mcfd.

Solution

Time of Stabilization

Substituting into equation (D-16)

$$t_s = \frac{(1000)(2640)^2(0.011)(0.20)}{(10)(625)}$$

$$t_s = 2450 \text{ hours}$$

Rate of Pressure Decline

From equation (D-14)

$$\frac{dP_t}{dt_D} = 2\left(\frac{0.25}{2640}\right)^2 = 1.79 \times 10^{-8}$$

Combining this with equation (D-12), we can determine the rate at which the sandface pressure will be declining when the flow conditions are semi-stabilized.

$$1.79 \times 10^{-8} = \frac{5.356 \times 10^{-3}\ (0.20)(0.25^2)(20)}{(0.90)(520)(0.5)} \left(- \frac{dP_s}{dt} \right)$$

$$- \frac{dP_s}{dt} = 0.0031 \text{ psi/hour}$$

So the sandface pressure will be declining at a very low rate when stabilization occurs and such a rate would have to be specified to result in stabilization equivalent to that attained after 2450 hours.

Example D-2

Calculate the time of stabilization for a high pressure, high permeability well, given the following reservoir data.

P_f = 2700 psia.

T_a = 640°R.

k = 1000 md.

ϕ = 0.09

h = 120 ft.

G = 0.81

T_c = 416°R.

P_c = 672 psia.

r_s = 0.25 ft.

r_f = 2640 ft.

Z_a = 0.79 (at an estimated average reservoir pressure of 2690 psia.)

μ_a = 0.019 cp. (at an estimated average reservoir pressure of 2690 psia.)

The well is flowed at a constant rate of 30.0 MMcfd.

Solution

Time of Stabilization

Substituting into equation (D-16)

$$t_s = \frac{(1000)(2640^2)(0.019)(0.09)}{(1000)(2700)} = 4.4 \text{ hours}$$

Rate of Pressure Decline

From equation (D-14)

$$\frac{dP_t}{dt_D} = 2\left(\frac{0.25}{2640}\right)^2 = 1.79 \times 10^{-8}$$

Combining with equation (D-12)

$$1.79 \times 10^{-8} = \frac{5.356 \times 10^{-3}(0.09)(0.25^2)(120)}{(0.79)(640)(30)} \left(- \frac{dP_s}{dt}\right)$$

$$- \frac{dP_s}{dt} = 0.075 \text{ psi/hour}$$

It can be seen that if adequate stabilization is to reflect a set drainage radius, no one pressure decline rate can be used as a criterion.

Conclusions

These examples illustrate that the time to achieve this arbitrarily defined stabilization (to the edge of a one section spacing unit), as calculated from a theoretical approach may be very long. In fact, from Example D-2, one may conclude that if the defined criterion of stabilization must be attained on each flow point, a conventional back pressure test is virtually impractical to run in all but the most permeable reservoirs. This calculated time may be adjusted on the basis of experience, with the object of determining the time after which the change in the flow rate and pressure are inconsequential with respect to the effect on flow in the reservoir. It is recognized that attainment of this condition is not always practical and an arbitrarily selected "maximum time" to stabilization is desirable. This Board has chosen a time of fifteen days. This

means that where the calculated time of stabilization is greater than fifteen days, it is considered that by the fifteenth day the changes occurring as the radius of drainage continues to move outwards in the reservoir, will have little effect on the interpretation of a flow test, bearing in mind the uses to which the test results will be put. This does not necessarily mean that a flow point need be extended for the lesser of the calculated time to stabilization or fifteen days. However, where the time to stabilization is longer than the length of the extended flow period, the stabilized conditions should be calculated using the method outlined in Section 5 or any other acceptable approach.

APPENDIX E

THE THEORETICAL VALIDITY OF THE ISOCHRONAL METHODS OF TESTING GAS WELLS

The starting point in a discussion of the theoretical validity of the isochronal test, is the steady-state equation (C-15) for combined laminar-turbulent flow.

$$P_f^{\,2} - P_s^{\,2} = \frac{1.418 \times 10^6}{kh} \mu_a Z_a T_a Q \ln\left(\frac{r_a}{r_s}\right) + \frac{3.14 \times 10^{-6}}{h^2} \beta\, G\, Z_a T_a Q^2 \left(\frac{1}{r_s} - \frac{1}{r_a}\right) \qquad \text{(E-1)}$$

where

P_f = formation pressure, psia.

P_s = flowing sandface pressure, psia.

k = permeability, md.

h = pay, ft.

μ_a = average viscosity, cp.

T_a = average temperature, °R.

Z_a = average compressibility factor.

r_a = apparent radius of drainage, ft.

r_s = effective well bore radius, ft.

β = turbulence factor.

Q = gas flow rate, millions of cubic feet per day at 14.65 psia and 60°F.

Equation (E-1) may be approximated by the following equation

$$C' \left(P_f^{\,2} - P_s^{\,2}\right)^n = Q \ln\left(\frac{r_a}{r_s}\right) \qquad \text{(E-2)}$$

where 'n' depends upon the total contribution of the two terms on the right-hand side of equation (E-1) being unity when the second term makes

no contribution, and for example being 0.95 when the maximum contribution is some 20 percent. The 'n' in equation (E-2) is represented as being a constant. This is an approximation, as a perusal of equation (E-1) clearly shows it to be a variable. In fact, 'n' is a constant (equal to unity) only when there is zero turbulence.

For a series of flow rates, if each of the flow rates is extended to a condition of stabilization, the effective radius of drainage is the edge of the drainage area, and the apparent radius is a constant. Equation (E-2) may then be written as

$$C(P_f^2 - P_s^2)^n = Q \tag{E-3}$$

where

C = the stabilized back pressure coefficient.

Several authors (44)(45)(55) have shown that the effective radius of drainage for a particular reservoir is a function only of time. Since a definite relationship has been shown to exist between the effective radius of drainage and the apparent radius of drainage, it follows that for any series of flow rates carried out for a constant duration following a prolonged period of shut-in (an isochronal test), the apparent radius of drainage will be a constant and equation (E-2) may be expressed as

$$C''(P_f^2 - P_s^2)^n = Q \tag{E-4}$$

where C'' is some coefficient other than the stabilized coefficient.

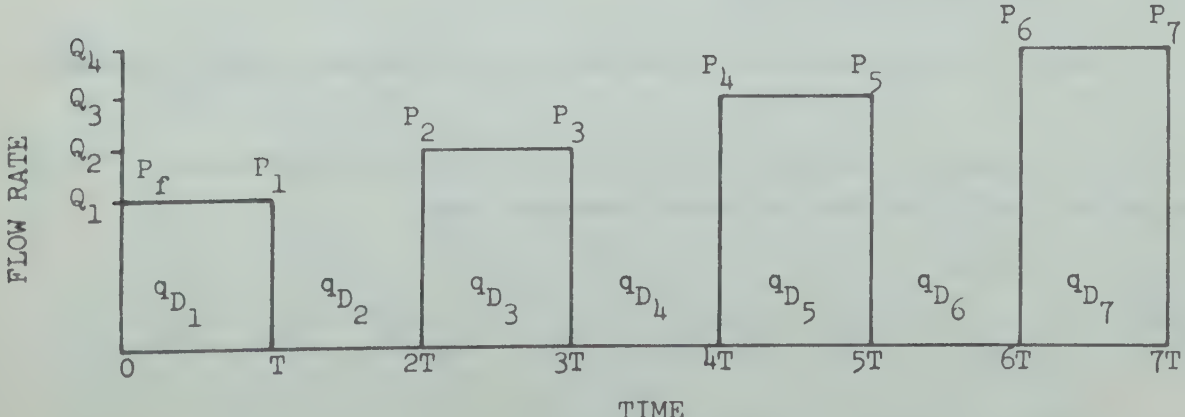
The turbulence term in equation (E-1) is essentially independent of the radius of drainage, r_a, and is the same for a short time isochronal flow period as it would be for a longer stabilized flow period. However, the laminar term (and thus C'' as designated in equation (E-4)) differs. As a result, the 'n' in equation (E-4) will differ from that of equation

(E-3) even though it corrects for the same amount of turbulence. This difference in the value of the exponent 'n' will depend on the degree of stabilization which exists at the time interval selected for conducting the isochronal test. Several calculations have indicated that the change in 'n' will be small, even in the extreme case where the drainage radius affected during the test is very small compared to the stabilized radius of drainage. Experience verifies this, and in fact indicates that in most instances, changes in the exponent 'n' as the flow period is extended in length, are immeasurable.

The results of an isochronal test may then be plotted as $(P_f^2 - P_s^2)$ versus Q and will result in a straight line with a slope such that the exponent 'n' is essentially the same as would be experienced under conditions of stabilized flow. If this isochronal line is positioned to reflect stabilized conditions, the results of a fully stabilized conventional back pressure test are duplicated.

Modified Isochronal Test

From unsteady-state flow theory it can be shown as follows that the results of a modified isochronal test are almost identical to those obtained from an isochronal test.



Isochronal Test

For the isochronal test the time between each flow period is not equal to T but is sufficient to permit the pressure to return to P_f.

Consider the final flow rate, Q_4. This is plotted against $P_f^2 - P_7^2$ where P_7 is the sandface pressure at the end of flow rate q_{D_7} (Q_4).

From equation (C-34)

$$\frac{P_f^2 - P_7^2}{P_f^2} = q_{D_7} P_{t_7}$$

P_{t_7} corresponds to the time since the 7th flow rate (q_{D_7}, Q_4) began.

From equation (C-36) $P_{t_7} = 1/2 (\ln t_{D_7} + 0.809)$

Assuming the effects of changes in pressure and viscosity can be neglected then $P_{t_7} = 1/2 (\ln \text{"constant"}\ T + 0.809)$.

Combining the constants into "a" and "b"

$$P_{t_7} = \ln bT + a$$

and

$$\frac{P_f^2 - P_7^2}{P_f^2} = q_{D_7} (\ln bT + a) \tag{1}$$

Modified Isochronal

In this case the time between each flow period is equal to T and P_2, P_4 and P_6 are not equal to P_f.

Again consider the final flow rate, Q_4. This is plotted against $P_6^2 - P_7^2$.

From equation (C-38) for the multiple transient case

$$\frac{P_f^2 - P_6^2}{P_f^2} = q_{D_1} P_{t_1} + (q_{D_2} - q_{D_1}) P_{t_2} + (q_{D_3} - q_{D_2}) P_{t_3} + (q_{D_4} - q_{D_3}) P_{t_4}$$

$$+ (q_{D_5} - q_{D_4}) P_{t_5} + (q_{D_6} - q_{D_5}) P_{t_6}$$

where $q_{D_2} = q_{D_4} = q_{D_6} = 0$

and P_{t_1} = time since first flow rate began = 6 T

P_{t_2} = time since second flow rate began = 5 T

P_{t_3} = time since third flow rate began = 4 T

P_{t_4} = time since fourth flow rate began = 3 T

P_{t_5} = time since fifth flow rate began = 2 T

P_{t_6} = time since sixth flow rate began = T

So $$\frac{P_f^2 - P_6^2}{P_f^2} = q_{D_1} (P_{t_1} - P_{t_2}) + q_{D_3} (P_{t_3} - P_{t_4}) + q_{D_5} (P_{t_5} - P_{t_6})$$

but $q_{D_1} (P_{t_1} - P_{t_2}) = q_{D_1} \left[\ln b\ (6T) + a - \ln b\ (5T) + a \right] = q_{D_1} \ln\left(\frac{6}{5}\right)$

Similarly $q_{D_3} (P_{t_3} - P_{t_4}) = q_{D_3} \ln\left(\frac{4}{3}\right)$ and $q_{D_5} (P_{t_5} - P_{t_6}) = q_{D_5} \ln\left(\frac{2}{1}\right)$

So $$\frac{P_f^2 - P_6^2}{P_f^2} = q_{D_1} \ln\left(\frac{6}{5}\right) + q_{D_3} \ln\left(\frac{4}{3}\right) + q_{D_5} \ln\left(\frac{2}{1}\right) \qquad (2)$$

Also from equation (C-38)

$$\frac{P_f^2 - P_7^2}{P_f^2} = q_{D_1} P_{t_1} + (q_{D_2} - q_{D_1}) P_{t_2} + (q_{D_3} - q_{D_2}) P_{t_3} + (q_{D_4} - q_{D_3}) P_{t_4} +$$

$$(q_{D_5} - q_{D_4}) P_{t_5} + (q_{D_6} - q_{D_5}) P_{t_6} + q_{D_7} P_{t_7}$$

where $q_{D_2} = q_{D_4} = q_{D_6} = 0$

and P_{t_1} now corresponds to time since first flow rate began or to

7T, P_{t_2} - 6T, P_{t_3} - 5T, P_{t_4} - 4T, P_{t_5} - 3T, P_{t_6} - 2T, P_{t_7} - T.

then $$\frac{P_f^{\,2} - P_7^{\,2}}{P_f^{\,2}} = q_{D_1} (P_{t_1} - P_{t_2}) + q_{D_3} (P_{t_3} - P_{t_4}) + q_{D_5} (P_{t_5} - P_{t_6}) + q_{D_7} P_{t_7}$$

$$\frac{P_f^{\,2} - P_7^{\,2}}{P_f^{\,2}} = q_{D_1} \ln\left(\frac{7}{6}\right) + q_{D_3} \ln\left(\frac{5}{4}\right) + q_{D_5} \ln\left(\frac{3}{2}\right) + q_{D_7} (\ln bT + a) \qquad (3)$$

Subtracting equation (2) from (3)

$$\frac{P_6^{\,2} - P_7^{\,2}}{P_f^{\,2}} = q_{D_1} \left(\ln \frac{7}{6} - \ln \frac{6}{5}\right) + q_{D_3} \left(\ln \frac{5}{4} - \ln \frac{4}{3}\right) + q_{D_5} \left(\ln \frac{3}{2} - \ln \frac{2}{1}\right)$$

$$+ q_{D_7} (\ln bT + a)$$

$$= q_{D_1} \ln\left(\frac{35}{36}\right) + q_{D_3} \ln\left(\frac{15}{16}\right) + q_{D_5} \ln\left(\frac{3}{4}\right) + q_{D_7} (\ln bT + a) \qquad (4)$$

Comparing equation (1), the isochronal test, and equation (4), the modified isochronal test, the difference in the fourth point, flow rate Q_4 is:

$$P_f^{\,2} \left[q_{D_1} \ln\left(\frac{35}{36}\right) + q_{D_3} \ln\left(\frac{15}{16}\right) + q_{D_5} \ln\left(\frac{3}{4}\right)\right]$$

Similarly, it can be shown that the third point will differ by

$$P_f^{\,2} \left[q_{D_1} \ln\left(\frac{15}{16}\right) + q_{D_3} \ln\left(\frac{3}{4}\right)\right]$$

and the second point will differ by

$$P_f^{\,2} \left[q_{D_1} \ln\left(\frac{3}{4}\right)\right]$$

The first point will be identical.

The validity of the modified isochronal test may also be illustrated by a numerical example as follows, showing that the apparent radius of drainage will be nearly the same for a series of flow rates of equal duration, even though each flow rate does not begin at the same pressure.

Example E-1

A new well drilled in a reservoir with a virgin pressure of 1000 psia (from which no production has taken place), is flowed at a constant rate of

1.0 MMcfd for a period of three hours. The well is then closed in and flowed for alternate three hour periods. The second flow rate is at 1.5 MMcfd, the third is at 2.0 MMcfd, and the fourth flow rate is at 3.0 MMcfd. Calculate the apparent radius of drainage at the end of each of the flow periods. The following well data is given.

T_a = 550°R.

k = 30 md.

ϕ = 0.20

h = 20 ft.

G = 0.6

T_c = 360°R.

P_c = 672 psia.

r_s = 0.25 ft.

Z_a = 0.86

μ_a = 0.013 cp.

Solution

The flowing sandface pressures would normally be measured, but in this example they will be calculated from the unsteady-state flow equation. The example calculation assumes laminar flow.

(a) First Flow Rate

First trial

Assume P_s = 920 psia

$$P_a = \frac{1000 + 920}{2} = 960 \text{ psia}$$

From equation (C-32), the dimensionless time which corresponds to three hours is calculated.

$$t_D = \frac{(2.634 \times 10^{-4})(30)(960)(3)}{(0.013)(0.2)(0.25^2)} = 1.40 \times 10^5$$

For a short flow period such as three hours, the reservoir is assumed to act as an infinite reservoir, so from Figure C-2

$$P_t = 6.32$$

The dimensionless flow rate which corresponds to 1.0 MMcfd is calculated from equation (C-33).

$$q_D = \frac{(1.418 \times 10^6)(0.013)(0.86)(550)(1.0)}{(20)(30)(1000^2)} = 0.0145$$

Substituting into equation (C-34)

$$6.3 = \frac{1}{0.0145} \quad \frac{(1000^2 - P_s^{\,2})}{1000^2}$$

$$P_s = 953 \text{ psia}$$

Second trial

Assume P_s = 953 psia

$$P_a = \frac{1000 + 953}{2} = 977 \text{ psia}$$

From equation (C-32)

$$t_D = \frac{(2.634 \times 10^{-4})(30)(977)(3)}{(0.013)(0.2)(0.25^2)} = 1.43 \times 10^5$$

From Figure C-2

$$P_t = 6.33$$

$$q_D = 0.0145$$

Substituting into equation (C-34)

$$6.33 = \frac{1}{0.0145} \quad \frac{(1000^2 - P_s^{\,2})}{1000^2}$$

$$P_s = 953 \text{ psia}$$

So the sandface pressure after three hours will be 953 psia.

From the steady-state flow equation (C-8), the apparent radius of drainage may then be calculated.

$$1.0 = \frac{(0.7054 \times 10^{-6})(20)(30)(1000^2 - 953^2)}{(0.013)(0.86)(550)\ \ln(r_a/0.25)}$$

$$r_a = 138 \text{ ft.}$$

(b) Second Flow Rate

The pressure at the end of the second flow rate can be calculated from equation (C-38), for the multiple transient case.

$$\frac{P_f^{\,2} - P_s^{\,2}}{P_f^{\,2}} = q_{D_1} P_{t_1} + (q_{D_2} - q_{D_1}) P_{t_2} + (q_{D_3} - q_{D_2}) P_{t_3}$$

$$t_{D_1} = \frac{(2.634 \times 10^{-4})(30)(977)(9)}{(0.013)(0.2)(0.25^2)} = 4.28 \times 10^5$$

From Figure C-2

$$P_{t_1} = 6.87$$

$$q_{D_1} = \frac{(1.418 \times 10^6)(0.013)(0.86)(550)(1.0)}{(20)(30)(1000^2)} = 0.0145$$

$$t_{D_2} = 2.86 \times 10^5$$

$$P_{t_2} = 6.68$$

$$q_{D_2} = 0$$

$$t_{D_3} = 1.43 \times 10^5$$

$$P_{t_3} = 6.33$$

$$q_{D_3} = 0.0218$$

$$\frac{1000^2 - P_s^{\,2}}{1000^2} = (0.0145)(6.87) - (0.0145)(6.68) + (0.0218)(6.33)$$

$$P_s = 927 \text{ psia}$$

The sandface pressure at the end of the second flow period is 927 psia. The radius of drainage at this time can be calculated from equation (C-8), if the pressure is known at the start of the flow period.

This pressure (to which the well will be built up during the three hour shut-in), is also determined from the multiple transient flow equation.

$$\frac{1000^2 - P_s^{\,2}}{1000^2} = (0.0145)(6.68) - (0.0145)(6.33)$$

$$P_s = 997 \text{ psia}$$

So from equation (C-8)

$$1.5 = \frac{(0.7054 \times 10^{-6})(20)(30)(997^2 - 927^2)}{(0.013)(0.86)(550)\ \ln(r_a/0.25)}$$

$$r_a = 121 \text{ ft.}$$

(c) Third Flow Rate

The pressure at the end of the second shut-in period is then calculated from equation (C-38).

$$t_{D_1} \text{ (for 12 hours)} = 5.72 \times 10^5$$

$$P_{t_1} = 7.02$$

$$q_{D_4} = 0$$

$$\frac{1000^2 - P_s^{\,2}}{1000^2} = (0.0145)(7.02) - (0.0145)(6.87) + (0.0218)(6.68) - (0.0218)(6.33)$$

$$P_s = 995 \text{ psia}$$

The pressure after the third flow period is also calculated from equation (C-38).

$$t_{D_1} \text{ (for 15 hours)} = 7.15 \times 10^5$$

$$P_{t_1} = 7.14$$

$$q_{D_5} = 0.0290$$

$$\frac{1000^2 - P_s^{\,2}}{1000^2} = (0.0145)(7.14) - (0.0145)(7.02) + (0.0218)(6.87) - (0.0218)(6.68) + (0.0290)(6.33)$$

$$P_s = 900 \text{ psia}$$

from equation (C-8)

$$2.0 = \frac{(0.7054 \times 10^{-6})(20)(30)(995^2 - 900^2)}{(0.013)(0.86)(550)\ \ln(r_a/0.25)}$$

$$r_a = 123 \text{ ft.}$$

(d) Fourth Flow Rate

The pressure at the end of the third shut-in period is calculated from equation (C-38).

$$t_{D_1} \text{ (for 18 hours)} = 8.58 \times 10^5$$

$$P_{t_1} = 7.23$$

$$q_{D_6} = 0$$

$$\frac{1000^2 - P_s^{\,2}}{1000^2} = (0.0145)(7.23) - (0.0145)(7.14) + (0.0218)(7.02) - (0.0218)(6.87) + (0.0290)(6.68) - (0.0290)(6.33)$$

$$P_s = 993 \text{ psia}$$

The pressure after the third flow period is also calculated from equation (C-38).

$$t_{D_1} \text{ (for 21 hours)} = 1.00 \times 10^6$$

$$P_{t_1} = 7.31$$

$$q_{D_7} = 0.0435$$

$$\frac{1000^2 - P_s^{\,2}}{1000^2} = (0.0145)(7.31) - (0.0145)(7.23) + (0.0218)(7.14) - (0.0218)(7.02) + (0.0290)(6.87) - (0.0290)(6.68) + (0.0435)(6.33)$$

$$P_s = 846 \text{ psia}$$

From equation (C-8)

$$3.0 = \frac{(0.7054 \times 10^{-6})(20)(30)(993^2 - 846^2)}{(0.013)(0.86)(550)\ \ln(r_a/0.25)}$$

$$r_a = 123 \text{ ft.}$$

It can be seen for this particular reservoir that the apparent radius of drainage after three hours at any of the four flow rates is nearly the same, even though the formation pressure prior to each rate is not the same. This helps to justify the contention that the modified isochronal test yields results close to those of a true isochronal test.

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